ID2202 Lecture 07
Instruction Selection

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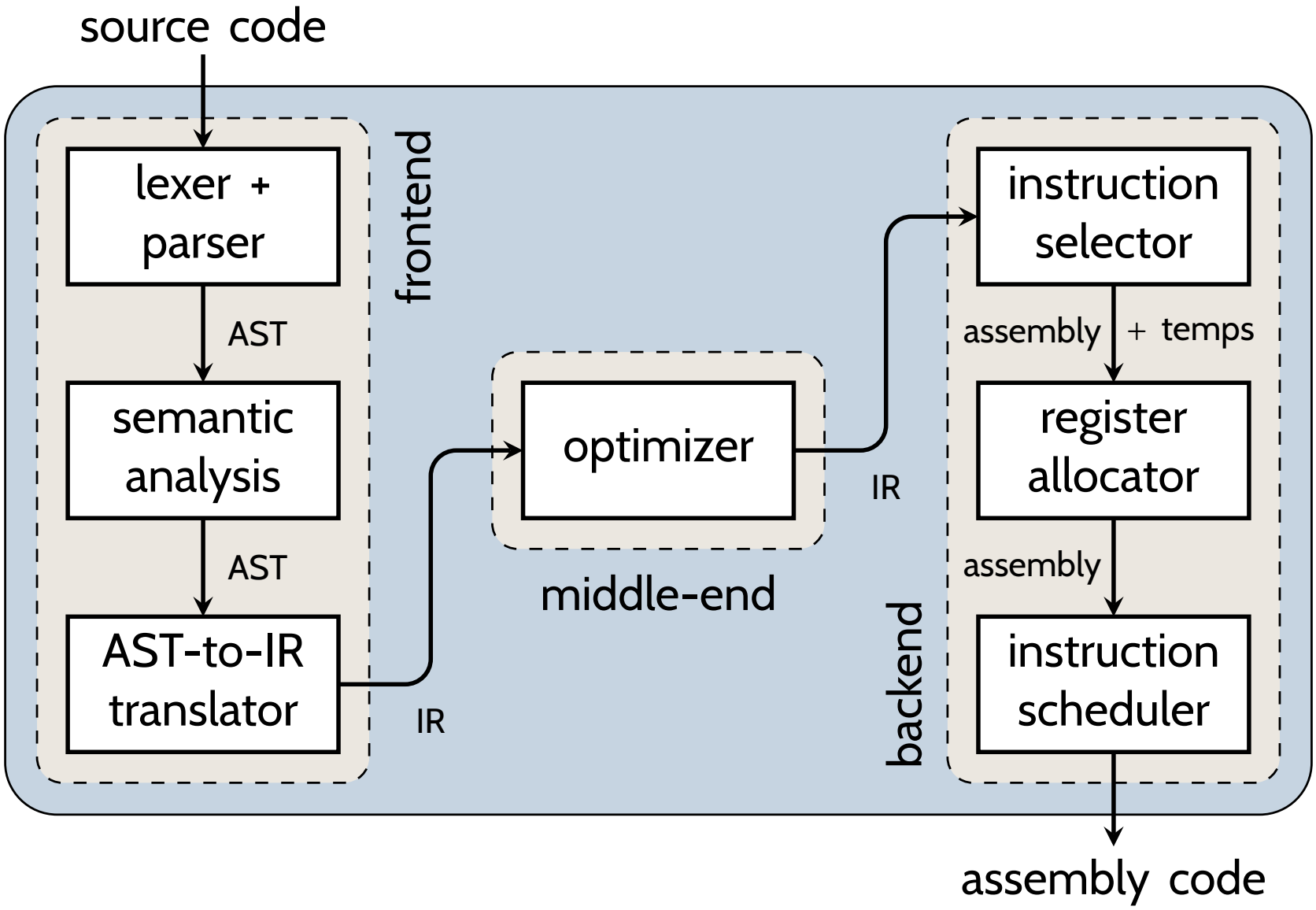
Compilation stages



IR (Intermediate Representation)

Using terminology from Tiger book:

- IR code consists of a list of **basic blocks**
- Each basic block contains a list of **statements**
  - First statement is LABEL,
  - Last statement is either JUMP or CJUMP,
  - All other statements are either MOVEs or EXPs
- Every statement shaped like an **IR tree**

Task of instruction selection

To translate each IR tree into corresponding sequence of assembly instructions

As running example for rest of lecture . . .

```
int  a[ ];
int  b[ ];
    ⋮
int  num = . . .
for  ( int  i  =  0;  i  <  num; i++)
{
   b[i ]  =  a[i ];
}
```

. . . will only consider this statement

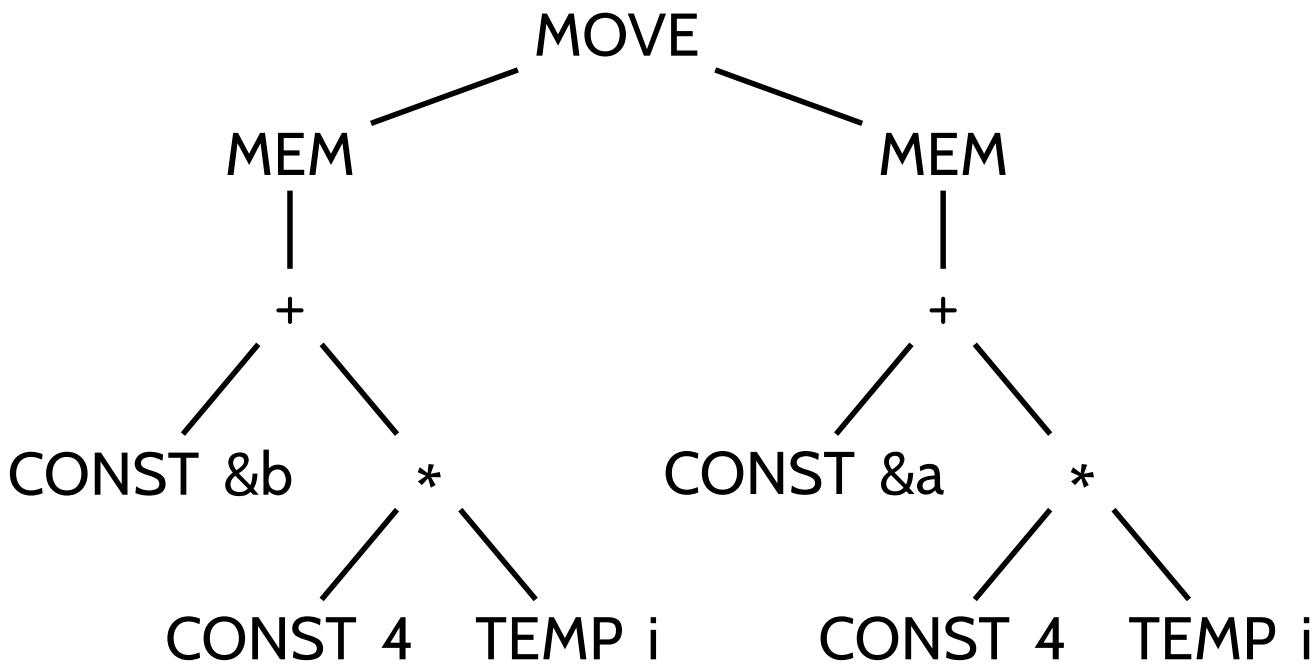
```
int a[ ];
int b[ ];
    ⋮
int num = . . .
for (int i = 0; i < num; i++)
{
  b[i] = a[i];
}
```

IR tree of **b[i] = a[i];**

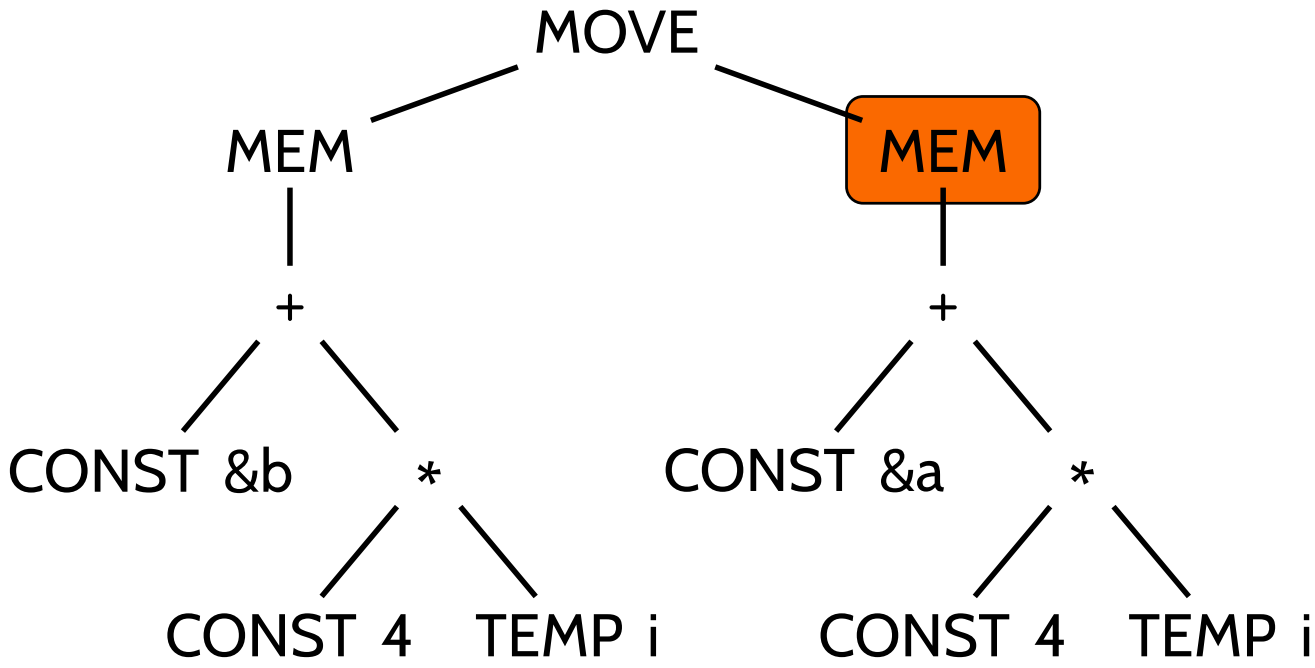
Assuming:

- Base memory address of **a** is CONST &a
- Base memory address of **b** is CONST &b
- Value of **i** is in TEMP i
- Size of **int** is CONST 4

IR tree of **b[i] = a[i];**

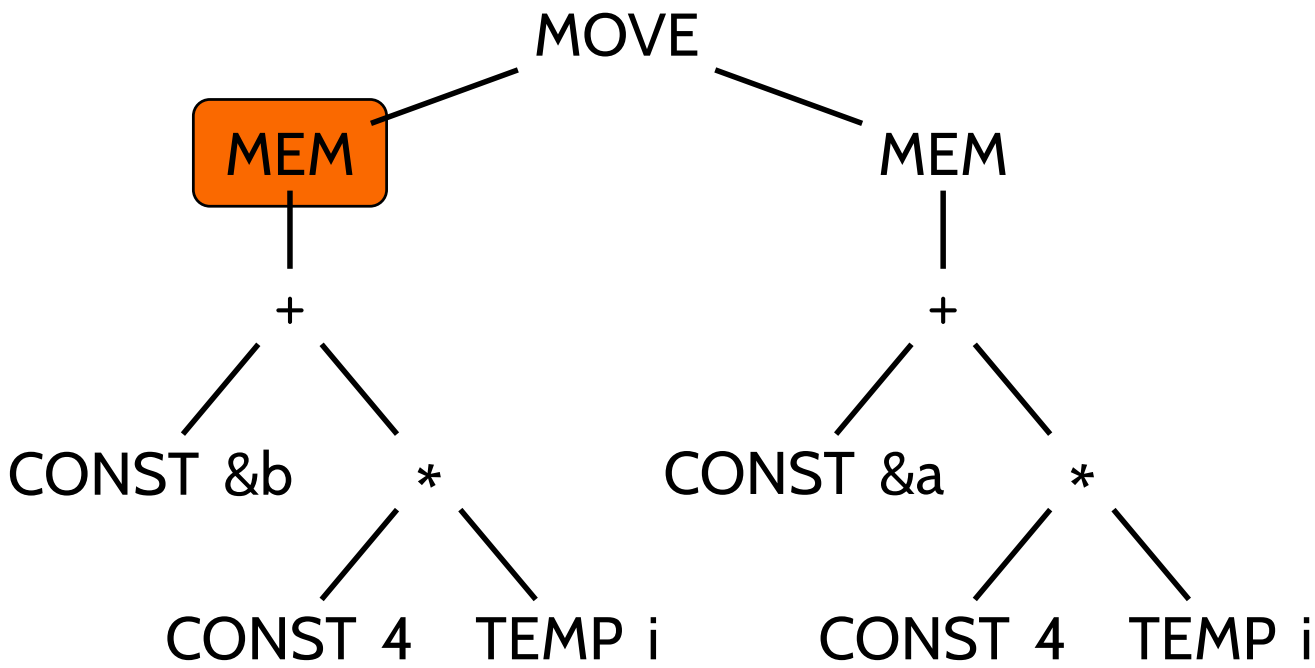


IR tree of **b[i] = a[i];**



MEM as **r-value** (right of MOVE) means "value of . . ."

IR tree of **b[i] = a[i];**



MEM as **l-value** (left of MOVE) means "location of . . ."

Target machine: *Jouette*

Notations:

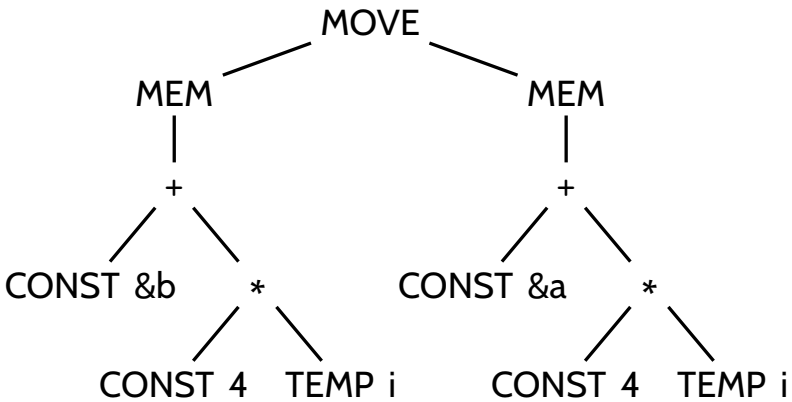
- r_i denotes "register *i*"
- t_i denotes "temporary *i*"
- *#c* denotes "integer constant *c*"
- *M*[*x*] denotes "memory value at address *x*"

Target machine: *Jouette*

Assembly instructions:		**Costs:**
ADD	$r_i \leftarrow r_j + r_k$	1
ADDI	$r_i \leftarrow r_j + \#c$	1
MUL	$r_i \leftarrow r_j * r_k$	2
LOAD	$r_i \leftarrow M[r_j + \#c]$	10
STORE	$M[r_i + \#c] \leftarrow r_j$	10
MOVEM	$M[r_i] \leftarrow M[r_j]$	12

- Jouette has more instructions (see Tiger book)
- r_0 is always contains value 0
- Cost could be number of cycles, code size, ...

Problem to solve



?

ADD	$r_i \leftarrow r_j + r_k$
ADDI	$r_i \leftarrow r_j + \#c$
MUL	$r_i \leftarrow r_j * r_k$
LOAD	$r_i \leftarrow M[r_j + \#c]$
STORE	$M[r_i + \#c] \leftarrow r_j$
MOVEM	$M[r_i] \leftarrow M[r_j]$

1st approach:

Macro Expansion

Fundamental idea

- Every IR operation has predefined set of semantics
- Every assembly instruction has predefined set of semantics
- Data flow via temporaries
- For each IR operation:
  - Emit sequence of assembly instructions with equivalent semantics
  - Emission done through **expansion macros**

Macro for CONST

expand(CONST *c*) =

t_x = getNewTemp()

emit("ADDI $t_x \leftarrow r_0 + \#c$")

setResultIsIn(t_x)

- What if *c* is 0?

Better macro for CONST

expand(CONST *c*) =

if *c* == 0 **then**

setResultIsIn(r_0)

else

t_x = getNewTemp()

emit("ADDI $t_x \leftarrow r_0$ + #*c*")

setResultIsIn(t_x)

endif

Macro for TEMP

expand(TEMP *t*) =

setResultIsIn(t_t)

Macros for + and *

expand(+ E_{lhs} E_{rhs}) =

t_{lhs} = getResultOf(E_{lhs})

t_{rhs} = getResultOf(E_{rhs})

t_x = getNewTemp()

emit("ADD $t_x \leftarrow t_{lhs} + t_{rhs}$")

setResultIsIn(t_x)

- Likewise implementation for *

Macro for MEM

expand(MEM *E*) =

if isRValue() **then**

t_x = getNewTemp()

t_y = getResultOf(*E*)

emit("LOAD $t_x \leftarrow M[t_y + \#0]$")

setResultIsIn(t_x)

else *is L-value*

setResultIsIn(getResultOf(*E*))

endif

Macros for MOVE

expand(MOVE (MEM E_{lhs}) E_{rhs}) =

t_x = getResultOf(E_{lhs})

t_y = getResultOf(E_{rhs})

emit("STORE M[t_x + #0] ← t_y")

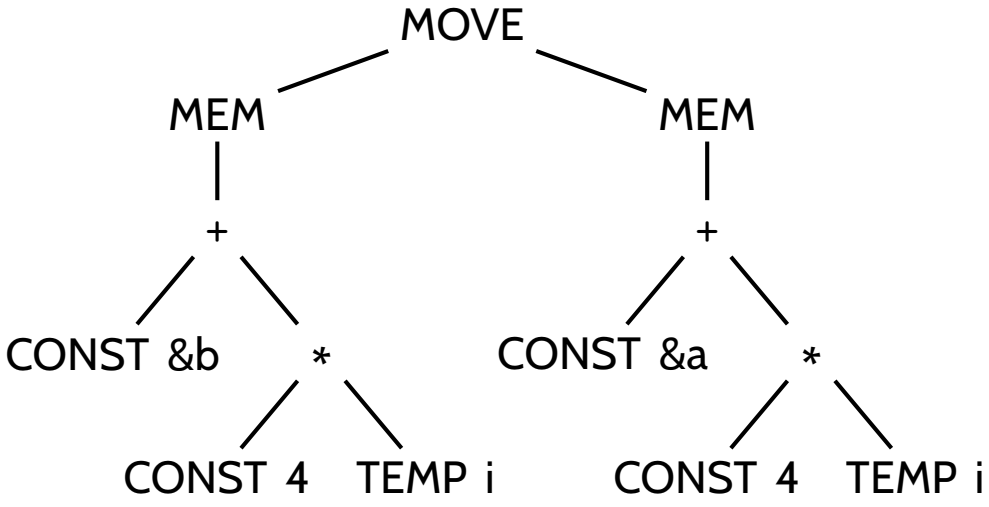
expand(MOVE E_{lhs} E_{rhs}) =

t_x = getResultOf(E_{lhs})

t_y = getResultOf(E_{rhs})

emit("ADD t_x ← r_0 + t_y")

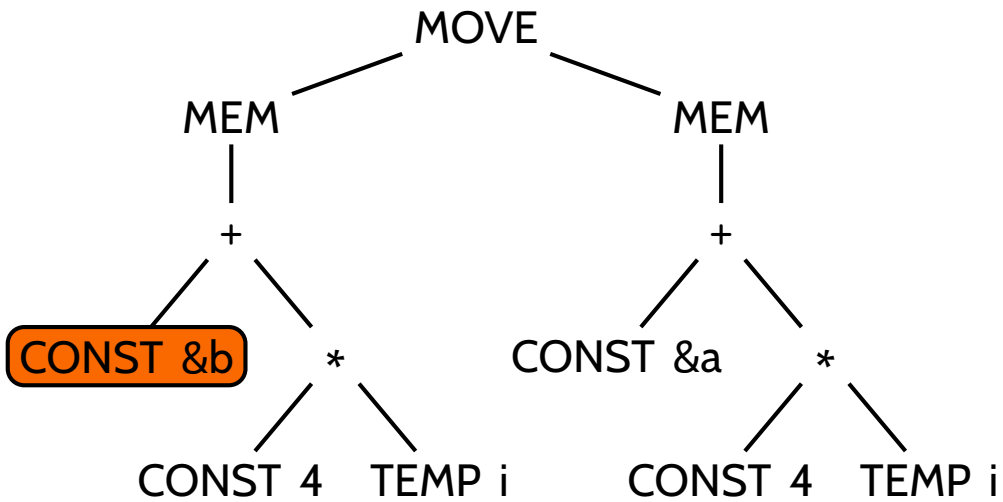
Running macro expansion on our IR tree



Assembly code:

Action:

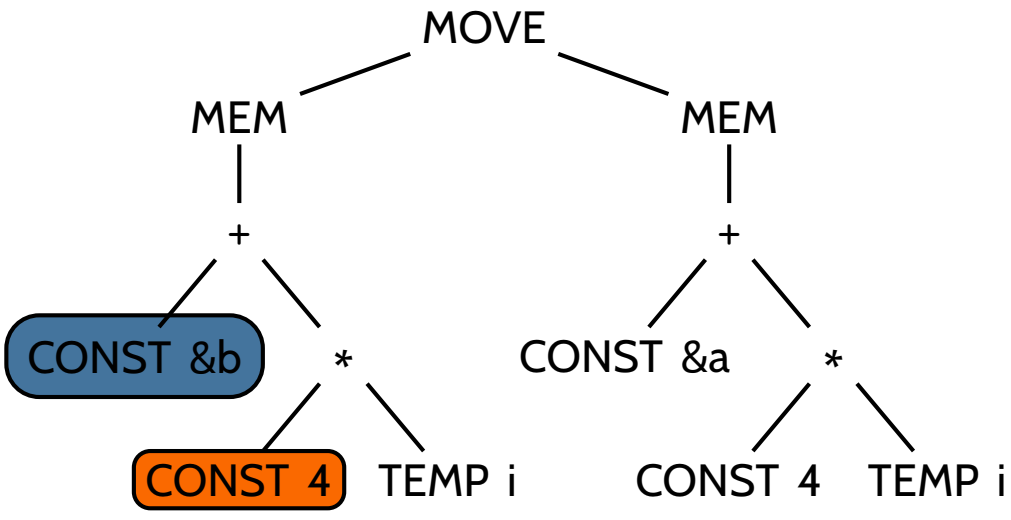
Running macro expansion on our IR tree



Assembly code:

Action: execute corresponding macro on each node

Running macro expansion on our IR tree

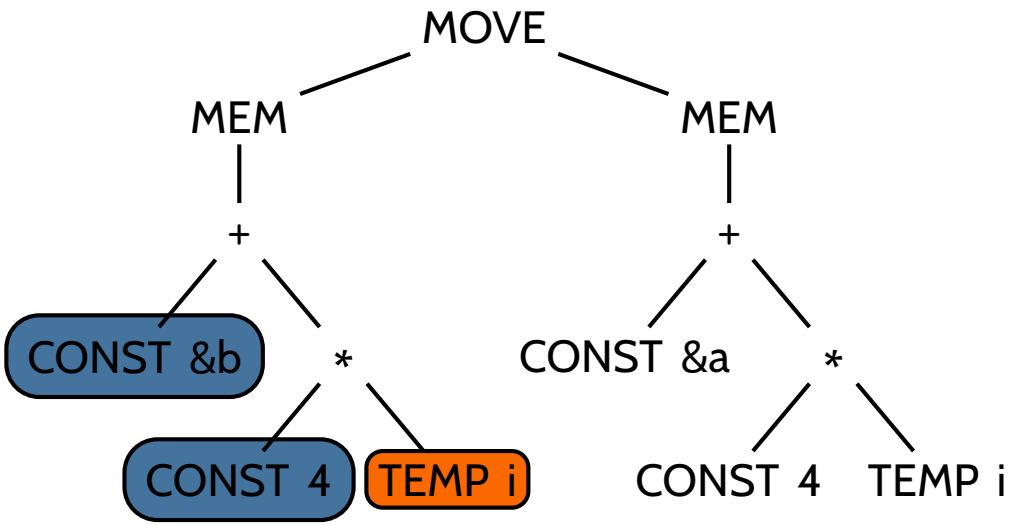


Assembly code:

ADDI $t_0 \leftarrow r_0 + \#\&b$

Action: execute corresponding macro on each node

Running macro expansion on our IR tree

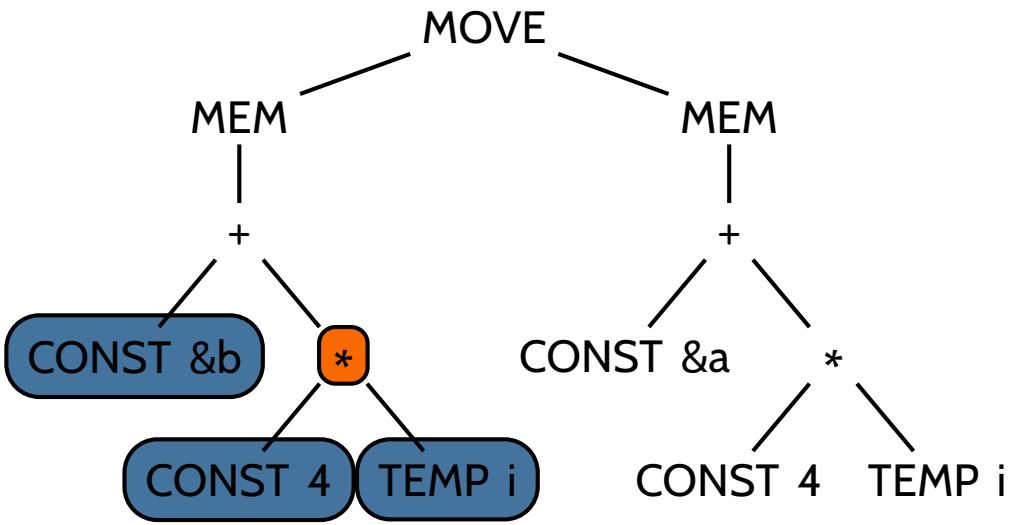


Assembly code:

ADDI $t_0 \leftarrow r_0 + \#\&b$
ADDI $t_1 \leftarrow r_0 + \#4$

Action: execute corresponding macro on each node

Running macro expansion on our IR tree

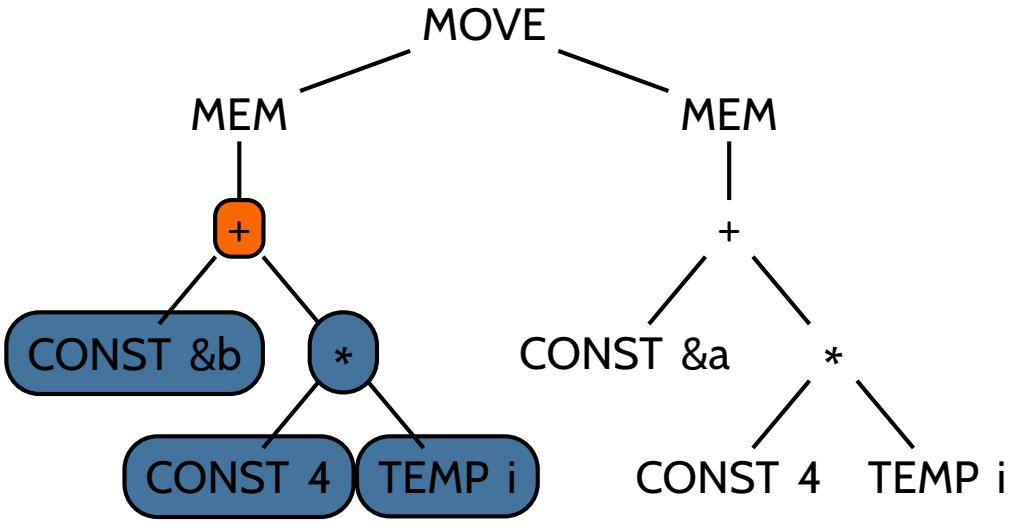


Assembly code:

ADDI $t_0 \leftarrow r_0 + \#\&b$
ADDI $t_1 \leftarrow r_0 + \#4$

Action: execute corresponding macro on each node

Running macro expansion on our IR tree

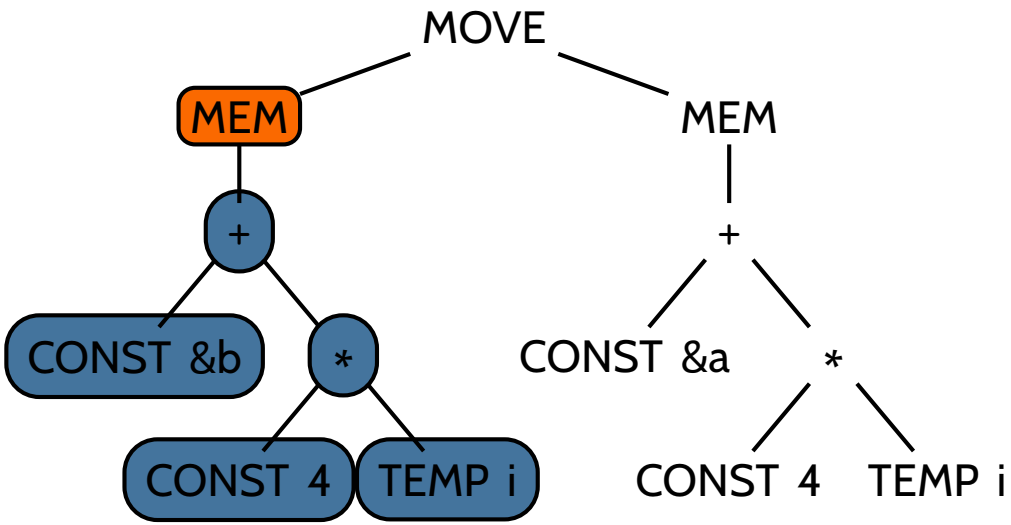


Assembly code:

```
ADDI    t0 ← r0 + #&b
ADDI    t1 ← r0 + #4
MUL     t2 ← t1 * ti
```

Action: execute corresponding macro on each node

Running macro expansion on our IR tree

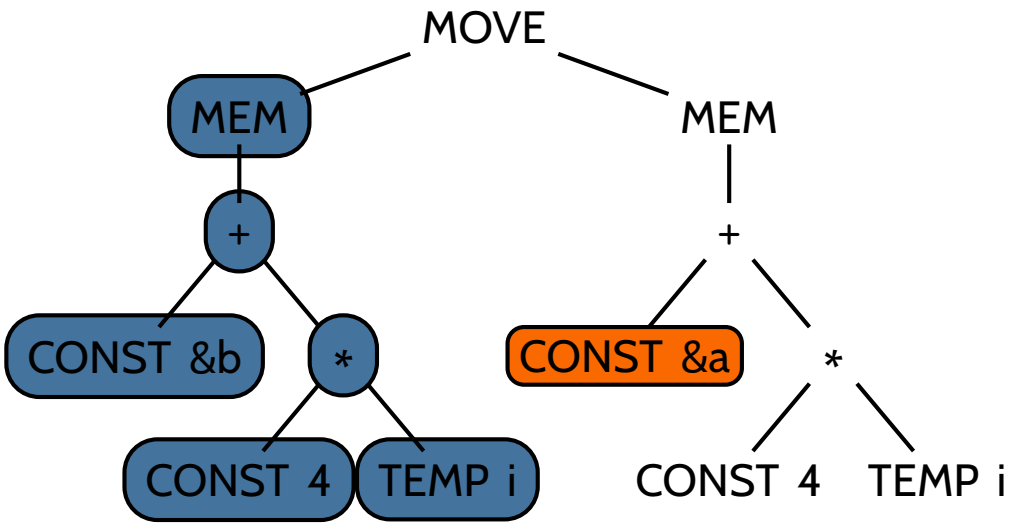


Assembly code:

```
ADDI   t0 ← r0 + #&b
ADDI   t1 ← r0 + #4
MUL    t2 ← t1 * ti
ADD    t3 ← t0 + t2
```

Action: execute corresponding macro on each node

Running macro expansion on our IR tree

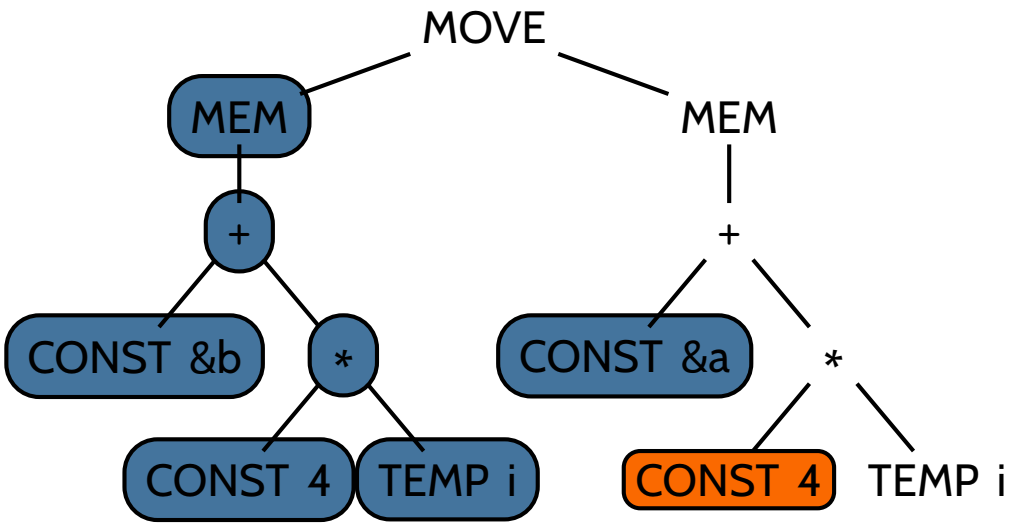


Assembly code:

```
ADDI   t0 ← r0 + #&b
ADDI   t1 ← r0 + #4
MUL    t2 ← t1 * ti
ADD    t3 ← t0 + t2
```

Action: execute corresponding macro on each node

Running macro expansion on our IR tree

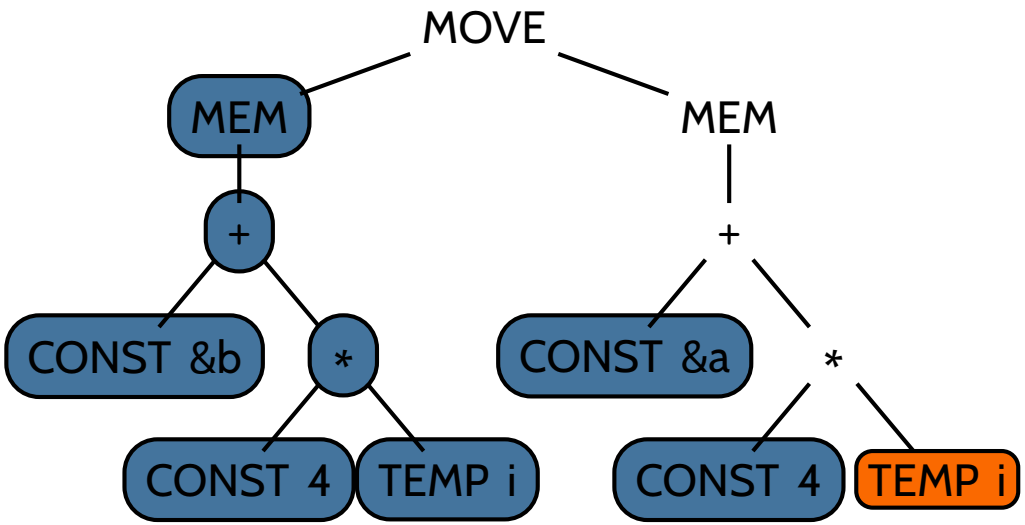


Assembly code:

```
ADDI   t0 ← r0 + #&b
ADDI   t1 ← r0 + #4
MUL    t2 ← t1 * ti
ADD    t3 ← t0 + t2
ADDI   t4 ← r0 + #&a
```

Action: execute corresponding macro on each node

Running macro expansion on our IR tree

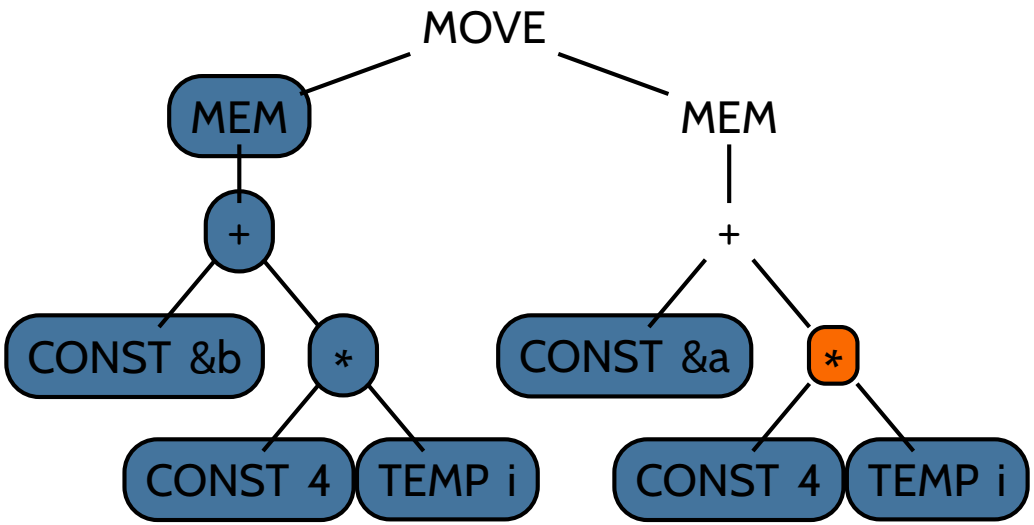


Assembly code:

```
ADDI   t0 ← r0 + #&b
ADDI   t1 ← r0 + #4
MUL    t2 ← t1 * ti
ADD    t3 ← t0 + t2
ADDI   t4 ← r0 + #&a
ADDI   t5 ← r0 + #4
```

Action: execute corresponding macro on each node

Running macro expansion on our IR tree

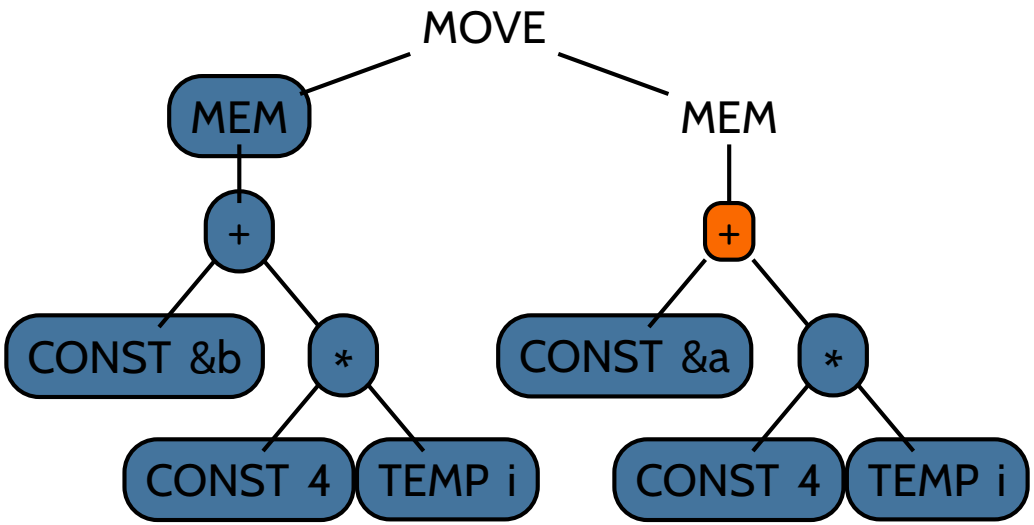


Assembly code:

```
ADDI   t0 ← r0 + #&b
ADDI   t1 ← r0 + #4
MUL    t2 ← t1 * ti
ADD    t3 ← t0 + t2
ADDI   t4 ← r0 + #&a
ADDI   t5 ← r0 + #4
```

Action: execute corresponding macro on each node

Running macro expansion on our IR tree

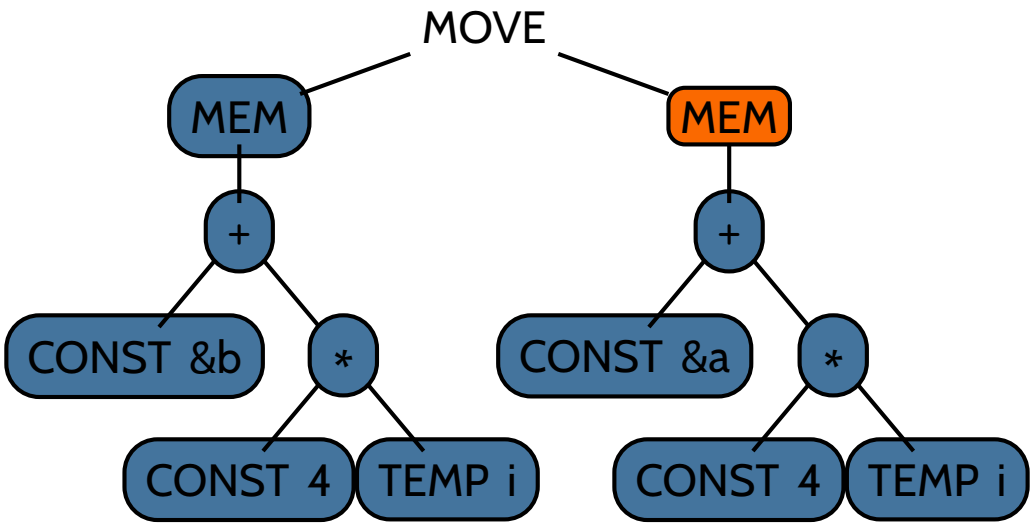


Assembly code:

```
ADDI   t0 ← r0 + #&b
ADDI   t1 ← r0 + #4
MUL    t2 ← t1 * ti
ADD    t3 ← t0 + t2
ADDI   t4 ← r0 + #&a
ADDI   t5 ← r0 + #4
MUL    t6 ← t5 * ti
```

Action: execute corresponding macro on each node

Running macro expansion on our IR tree

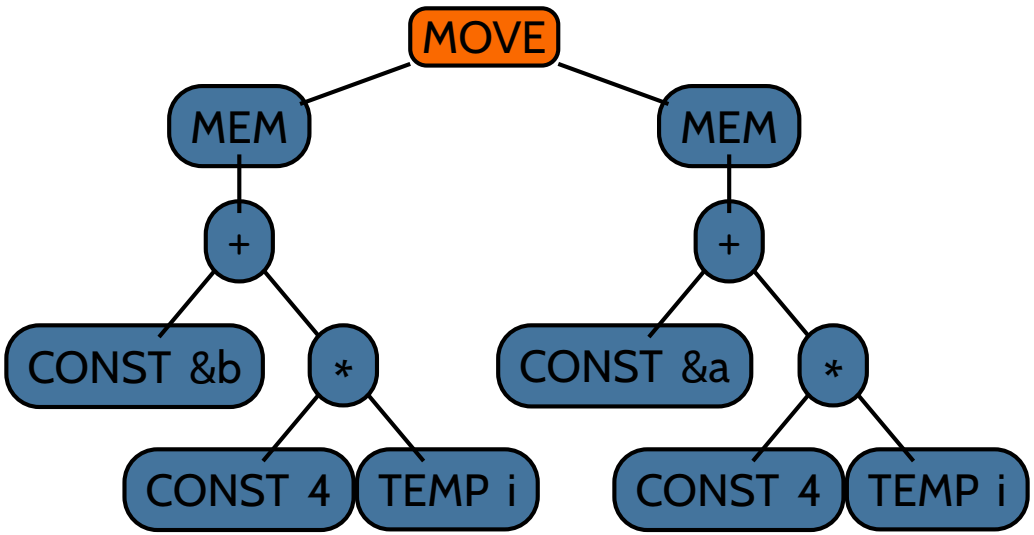


Assembly code:

```
ADDI   t0 ← r0 + #&b
ADDI   t1 ← r0 + #4
MUL    t2 ← t1 * ti
ADD    t3 ← t0 + t2
ADDI   t4 ← r0 + #&a
ADDI   t5 ← r0 + #4
MUL    t6 ← t5 * ti
ADD    t7 ← t4 + t6
```

Action: execute corresponding macro on each node

Running macro expansion on our IR tree

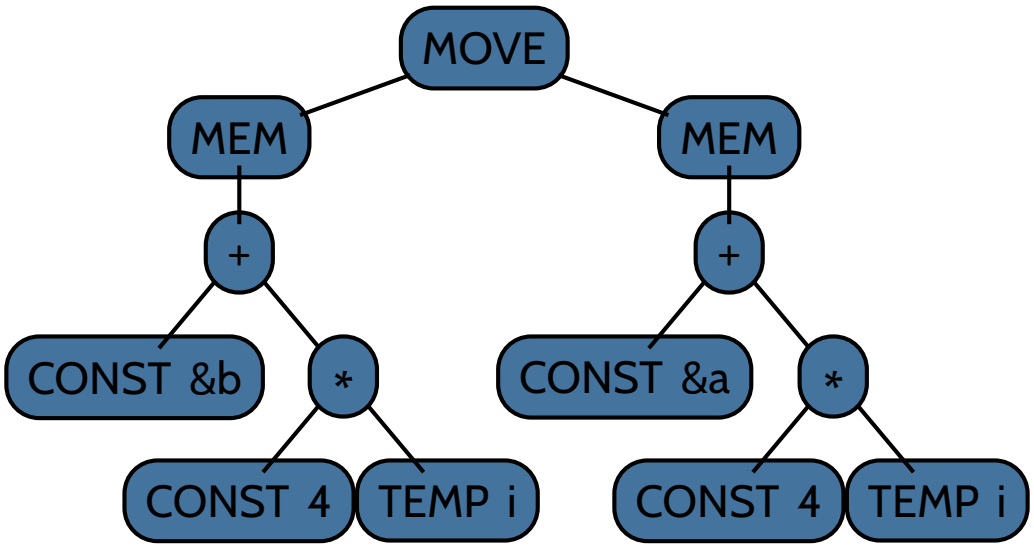


Assembly code:

```
ADDI   t0 ← r0 + #&b
ADDI   t1 ← r0 + #4
MUL    t2 ← t1 * ti
ADD    t3 ← t0 + t2
ADDI   t4 ← r0 + #&a
ADDI   t5 ← r0 + #4
MUL    t6 ← t5 * ti
ADD    t7 ← t4 + t6
LOAD   t8 ← M[t7 + #0]
```

Action: execute corresponding macro on each node

Running macro expansion on our IR tree



Assembly code:

```
ADDI    t0 ← r0 + #&b
ADDI    t1 ← r0 + #4
MUL     t2 ← t1 * ti
ADD     t3 ← t0 + t2
ADDI    t4 ← r0 + #&a
ADDI    t5 ← r0 + #4
MUL     t6 ← t5 * ti
ADD     t7 ← t4 + t6
LOAD    t8 ← M[t7 + #0]
STORE   M[t3 + #0] ← t8
```

Action: done

Quality of emitted assembly code

		Costs:
ADDI	$t_0 \leftarrow r_0 + \#\&b$	1
ADDI	$t_1 \leftarrow r_0 + \#4$	1
MUL	$t_2 \leftarrow t_1 * t_i$	2
ADD	$t_3 \leftarrow t_0 + t_2$	1
ADDI	$t_4 \leftarrow r_0 + \#\&a$	1
ADDI	$t_5 \leftarrow r_0 + \#4$	1
MUL	$t_6 \leftarrow t_5 * t_i$	2
ADD	$t_7 \leftarrow t_4 + t_6$	1
LOAD	$t_8 \leftarrow M[t_7 + \#0]$	10
STORE	$M[t_3 + \#0] \leftarrow t_8$	10

$$\sum \text{cost} = 30$$

Can we do better?

		Costs:
ADDI	$t_0 \leftarrow r_0 + \#\&b$	1
ADDI	$t_1 \leftarrow r_0 + \#4$	1
MUL	$t_2 \leftarrow t_1 * t_i$	2
ADD	$t_3 \leftarrow t_0 + t_2$	1
ADDI	$t_4 \leftarrow r_0 + \#\&a$	1
ADDI	$t_5 \leftarrow r_0 + \#4$	1
MUL	$t_6 \leftarrow t_5 * t_i$	2
ADD	$t_7 \leftarrow t_4 + t_6$	1
LOAD	$t_8 \leftarrow M[t_7 + \#0]$	10
STORE	$M[t_3 + \#0] \leftarrow t_8$	10

$$\sum \text{cost} = 30$$

Suggested improvement

		Costs:
ADDI	$t_0 \leftarrow r_0 + \#\&b$	1
ADDI	$t_1 \leftarrow r_0 + \#4$	1
MUL	$t_2 \leftarrow t_1 * t_i$	2
ADD	$t_3 \leftarrow t_0 + t_2$	1
ADDI	$t_4 \leftarrow r_0 + \#\&a$	1
ADDI	$t_5 \leftarrow r_0 + \#4$	1
MUL	$t_6 \leftarrow t_5 * t_i$	2
ADD	$t_7 \leftarrow t_4 + t_6$	1
LOAD	$t_8 \leftarrow M[t_7 + \#0]$	10
STORE	$M[t_3 + \#0] \leftarrow t_8$	10

$$\sum \text{cost} = 30$$

Result of improvement

		Costs:
ADDI	$t_0 \leftarrow r_0 + \#\&b$	1
ADDI	$t_1 \leftarrow r_0 + \#4$	1
MUL	$t_2 \leftarrow t_1 * t_i$	2
ADD	$t_3 \leftarrow t_0 + t_2$	1
ADDI	$t_5 \leftarrow r_0 + \#4$	1
MUL	$t_6 \leftarrow t_5 * t_i$	2
LOAD	$t_8 \leftarrow M[t_6 + \#\&a]$	10
STORE	$M[t_3 + \#0] \leftarrow t_8$	10

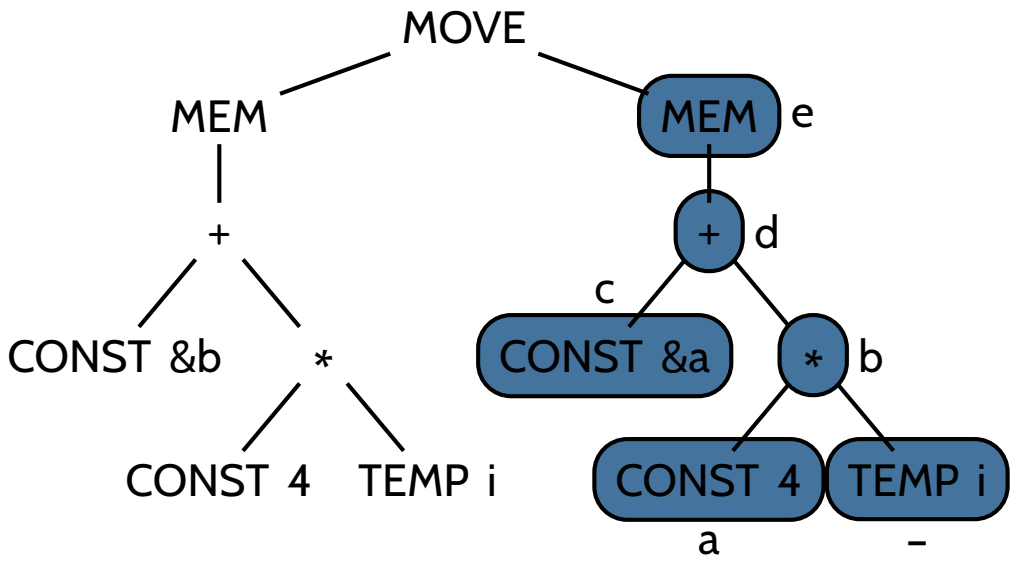
$$\sum \text{cost} = 28$$

Why did not macro expansion emit this?

		Costs:
ADDI	$t_0 \leftarrow r_0 + \#\&b$	1
ADDI	$t_1 \leftarrow r_0 + \#4$	1
MUL	$t_2 \leftarrow t_1 * t_i$	2
ADD	$t_3 \leftarrow t_0 + t_2$	1
ADDI	$t_5 \leftarrow r_0 + \#4$	1
MUL	$t_6 \leftarrow t_5 * t_i$	2
LOAD	$t_8 \leftarrow M[t_6 + \#\&a]$	10
STORE	$M[t_3 + \#0] \leftarrow t_8$	10

$$\sum \text{cost} = 28$$

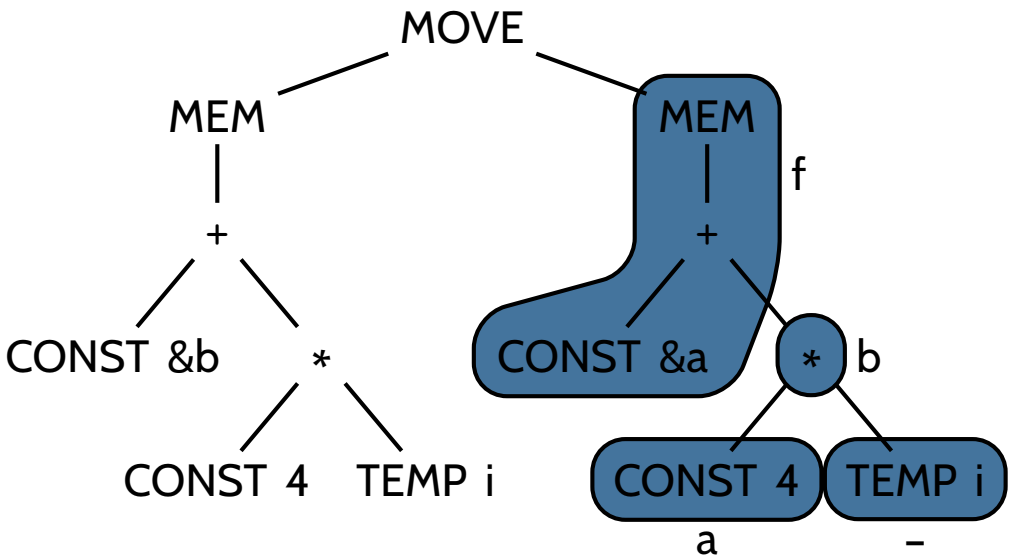
Limitation: *Macro expansion emits assembly code one IR operation at a time . . .*



Assembly code:

	ADDI	$t_0 \leftarrow r_0 + \#\&b$
	ADDI	$t_1 \leftarrow r_0 + \#4$
	MUL	$t_2 \leftarrow t_1 * t_i$
	ADD	$t_3 \leftarrow t_0 + t_2$
c	ADDI	$t_4 \leftarrow r_0 + \#\&a$
a	ADDI	$t_5 \leftarrow r_0 + \#4$
b	MUL	$t_6 \leftarrow t_5 * t_i$
d	ADD	$t_7 \leftarrow t_4 + t_6$
e	LOAD	$t_8 \leftarrow M[t_7 + \#0]$

. . . but some assembly instructions can implement *multiple* IR operations



Assembly code:

	ADDI	$t_0 \leftarrow r_0 + \#\&b$
	ADDI	$t_1 \leftarrow r_0 + \#4$
	MUL	$t_2 \leftarrow t_1 * t_i$
	ADD	$t_3 \leftarrow t_0 + t_2$
a	ADDI	$t_5 \leftarrow r_0 + \#4$
b	MUL	$t_6 \leftarrow t_5 * t_i$
f	LOAD	$t_8 \leftarrow M[t_6 + \#\&a]$

Instruction Selection as a Covering Problem

Let's refine the task of instruction selection

Before:

- To translate each IR tree into corresponding sequence of assembly instructions

Now:

- To **cover** each IR tree using set of **tree tiles** (often also called **tree patterns**), such that:
  - every IR operation is covered by exactly one tile (no tiles overlap)
- Tile set derived from instruction set
- Valid cover is called a **tiling**
- Prefer tiling T_1 over T_2 if

$$\sum_{p \in T_1} \text{cost}(p) < \sum_{p \in T_2} \text{cost}(p)$$

Optimal and optimum tilings

- **Optimal tiling:**
  - If two adjacent tiles cannot be combined into single tile with lower cost
  - Can be found using greedy target algorithms
  - Often sufficient for simple architectures
- **Optimum tiling:**
  - If tiling has least cost
  - Requires non-greedy algorithms
  - Beneficial when significant cost difference between optimum and optimal tilings

In literature only Tiger book uses these notions

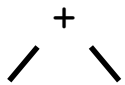
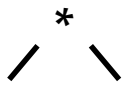
Subproblems to solve

- **Tile matching:**
  - Which tiles could cover what parts of the IR tree?
- **Tile selection:**
  - Which tiles to choose to form a tiling?
- **Optimality:**
  - How to find optimal/optimum tiling?

Revisiting macro expansion

- Requires all tiles to consist of single IR operation
  - Trivial to match tiles
- Existed only one tile per IR operation
  - Trivial to form tilings
- Can only find one tiling
  - Suboptimal by design

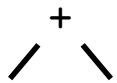
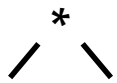
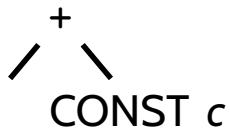
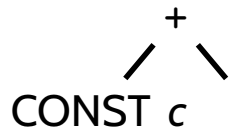
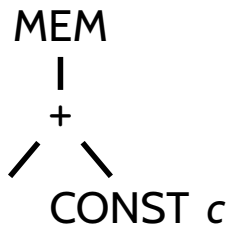
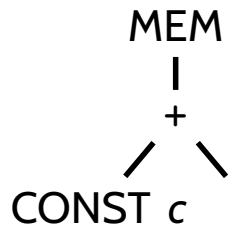
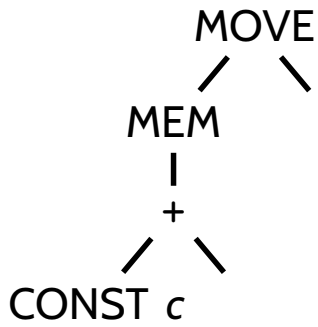
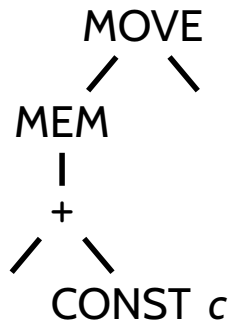
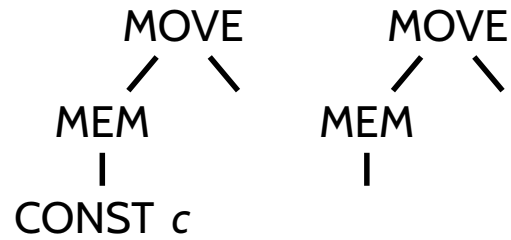
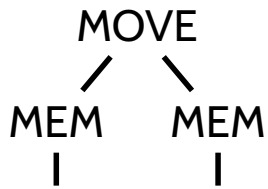
Tiles set of macro expansion

Tile	Instructions	
TEMP *c*	–	
CONST *c*	ADDI	$t_x \leftarrow r_0 + \#c$
	ADD	$t_x \leftarrow t_y + t_z$
	MUL	$t_x \leftarrow t_y * t_z$
	– *or* LOAD	 $t_x \leftarrow M[t_y + \#0]$
	STORE *or* ADD	$M[t_x + \#0] \leftarrow t_y$ $t_x \leftarrow r_0 + t_y$

Full tile set for our *Jouette* instructions

Instruction		Tiles
–		TEMP *c*
ADD	$t_x \leftarrow t_y + t_z$	+
MUL	$t_x \leftarrow t_y * t_z$	*
ADDI	$t_x \leftarrow t_y + \#c$	+ (CONST *c*) ; + (CONST *c*) ; CONST *c*
LOAD	$t_x \leftarrow M[t_y + \#c]$	MEM + (CONST *c*) ; MEM + (CONST *c*) ; MEM (CONST *c*) ; MEM
STORE	$M[t_x + \#c] \leftarrow t_y$	MOVE (MEM + (CONST *c*)) ; MOVE (MEM + (CONST *c*)) ; MOVE (MEM (CONST *c*)) ; MOVE (MEM)
MOVEM	$M[t_x] \leftarrow M[t_y]$	MOVE (MEM) (MEM)

Problem: How to use these efficiently?

Instruction		Tiles
–		TEMP *c*
ADD	$t_x \leftarrow t_y + t_z$	
MUL	$t_x \leftarrow t_y * t_z$	
ADDI	$t_x \leftarrow t_y + \#c$	  CONST *c*
LOAD	$t_x \leftarrow M[t_y + \#c]$	  
STORE	$M[t_x + \#c] \leftarrow t_y$	  
MOVEM	$M[t_x] \leftarrow M[t_y]$	

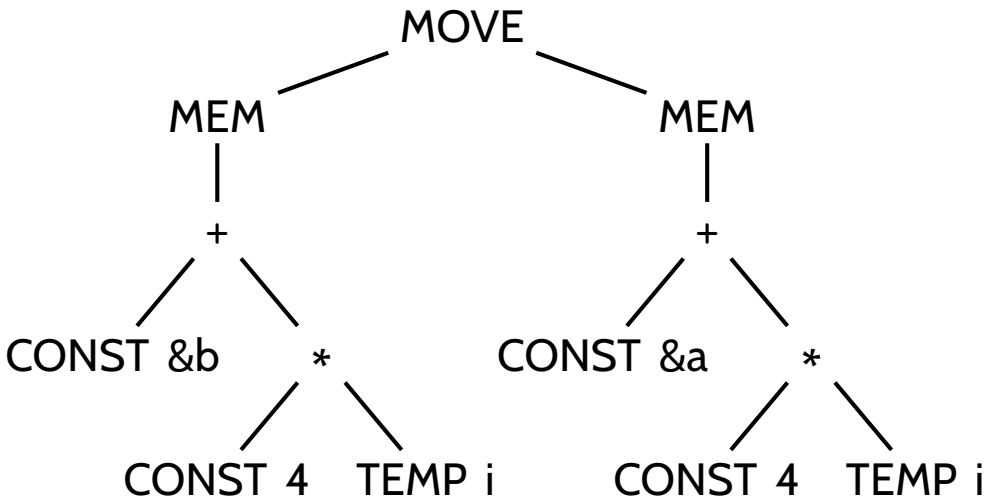
2nd approach:

Maximum Munch

Fundamental idea

- To find optimal tiling:
  1. Start at root node
  2. Find largest tile that matches at root
  3. Cover nodes matched by tile
  4. Repeat for all subtrees
- To emit assembly code:
  - Traverse IR tree bottom up
  - For each tile in tiling:
    - Emit corresponding instruction

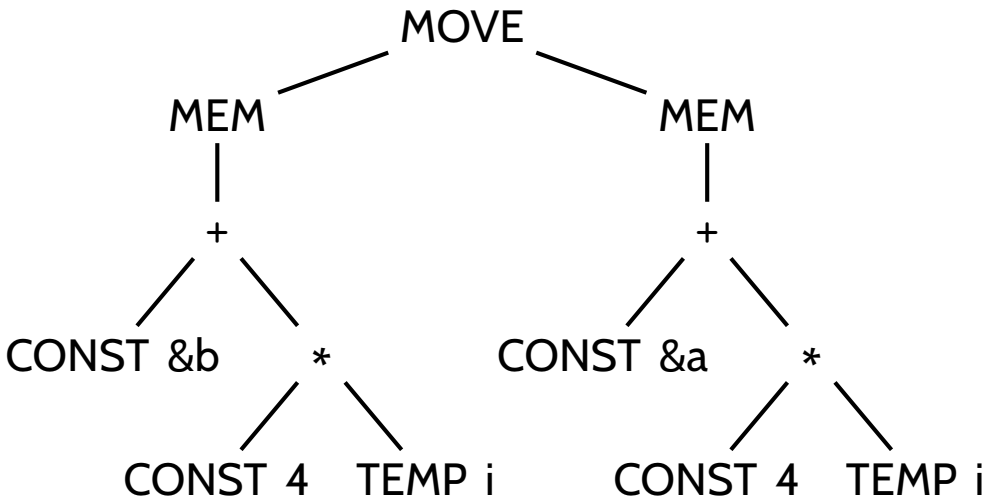
Running maximum munch on our IR tree



Assembly code:

Action:

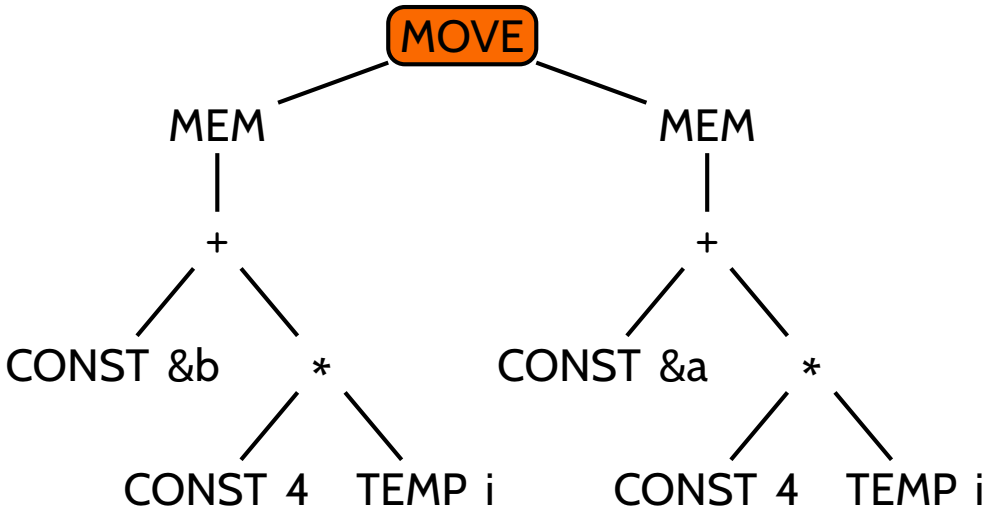
Running maximum munch on our IR tree



Assembly code:

Action: find largest matches

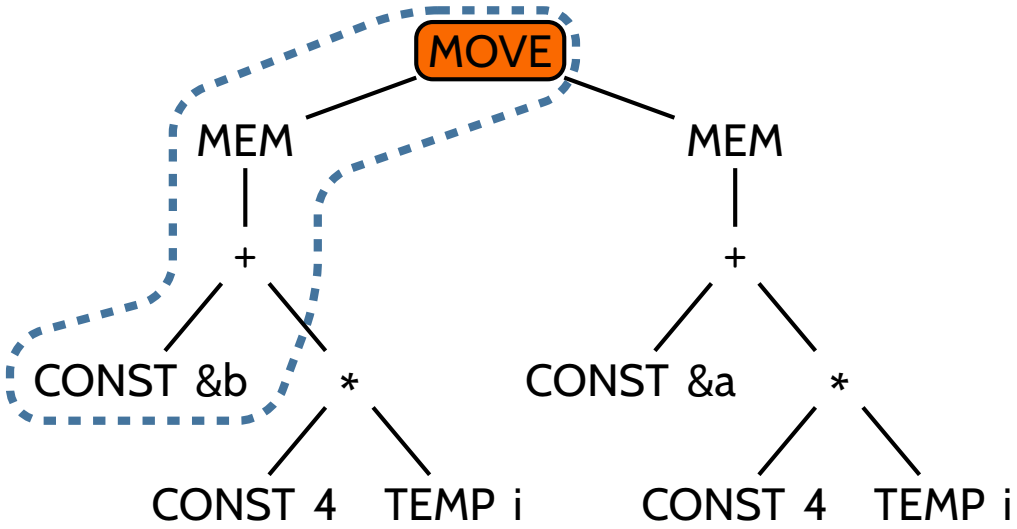
Running maximum munch on our IR tree



Assembly code:

Action: find largest matches

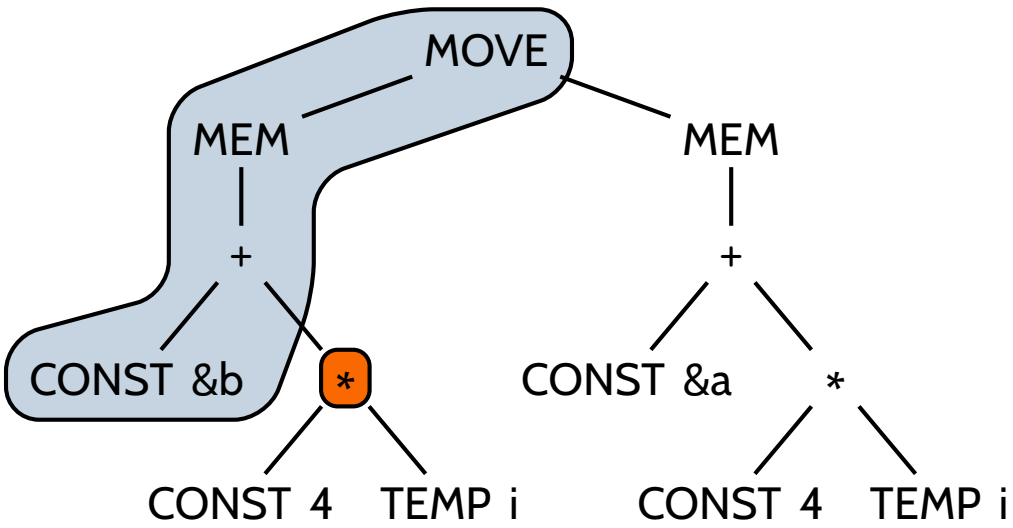
Running maximum munch on our IR tree



Assembly code:

Action: find largest matches

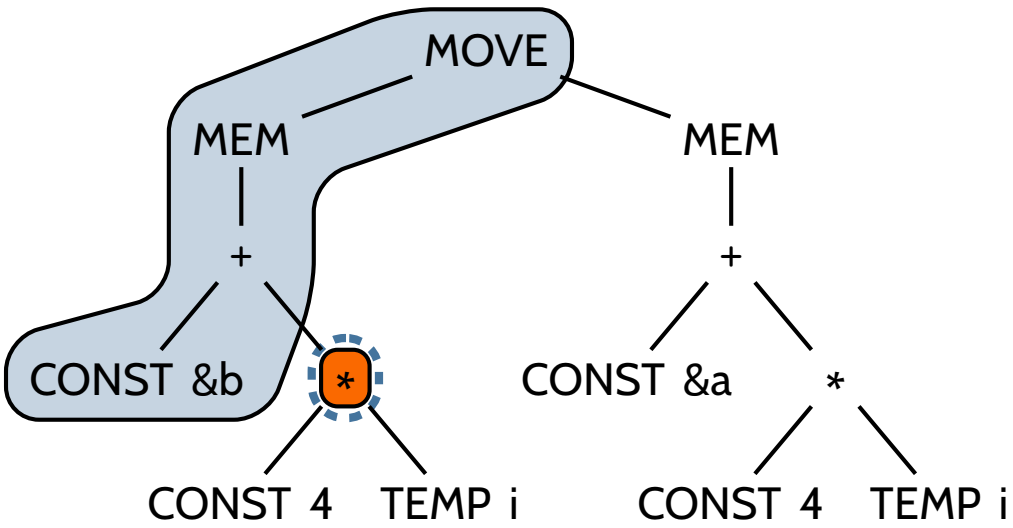
Running maximum munch on our IR tree



Assembly code:

Action: find largest matches

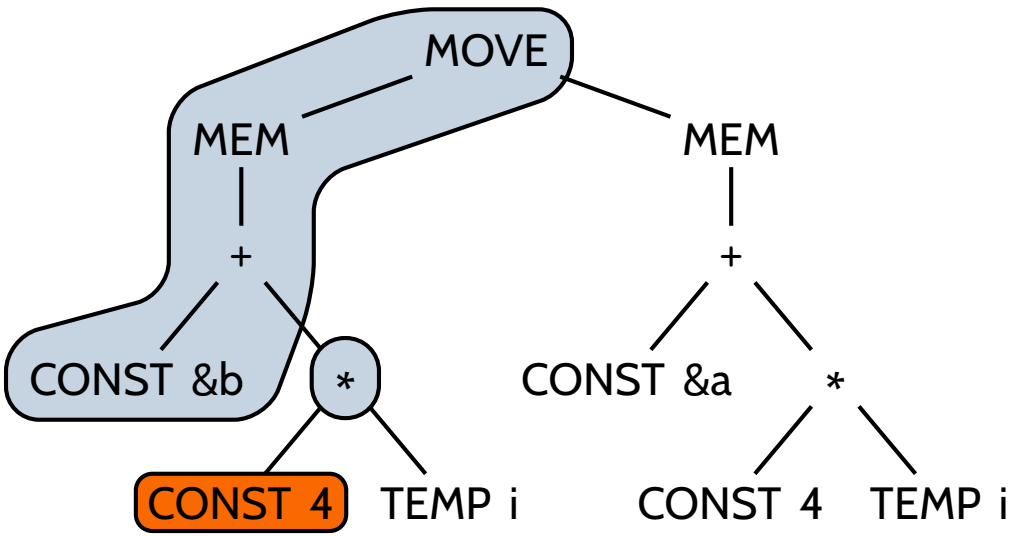
Running maximum munch on our IR tree



Assembly code:

Action: find largest matches

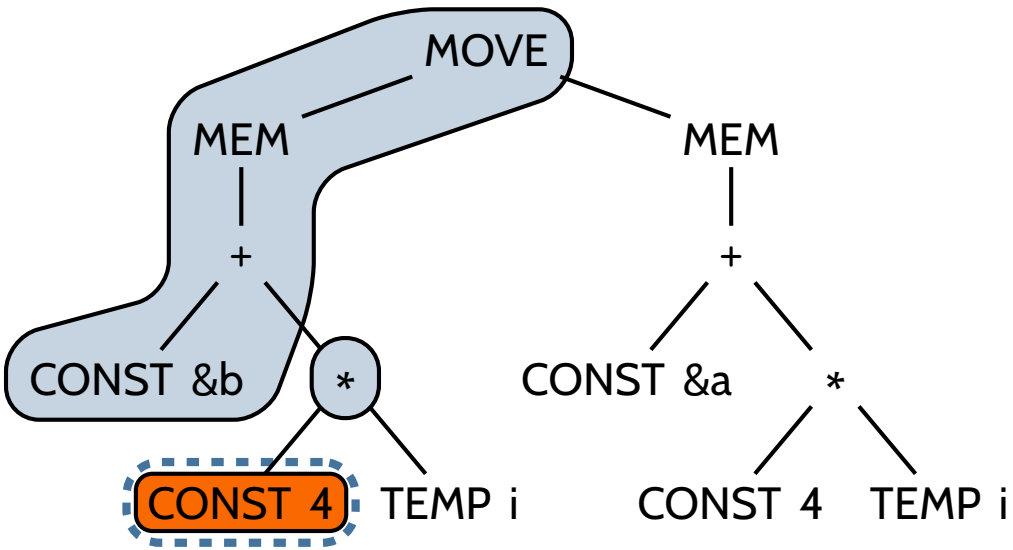
Running maximum munch on our IR tree



Assembly code:

Action: find largest matches

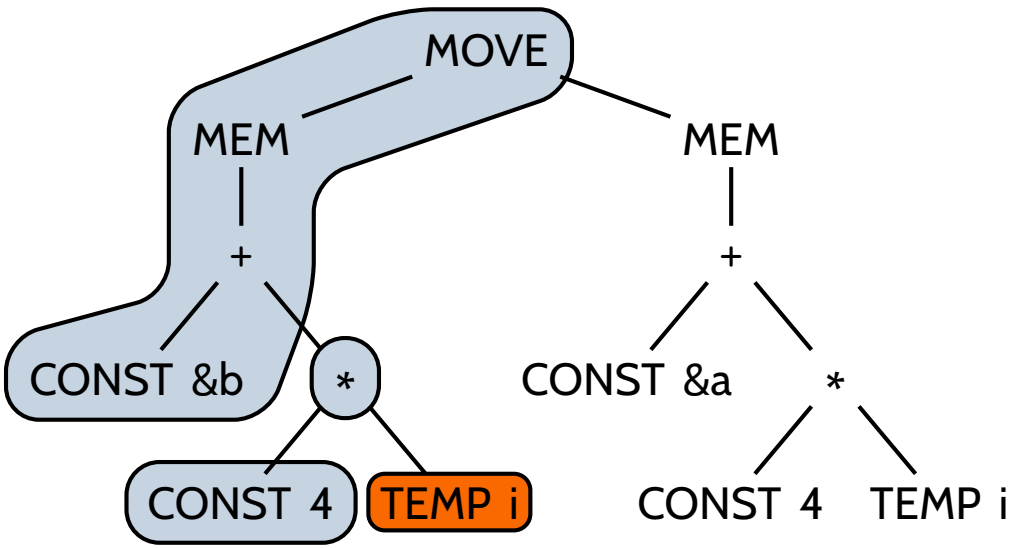
Running maximum munch on our IR tree



Assembly code:

Action: find largest matches

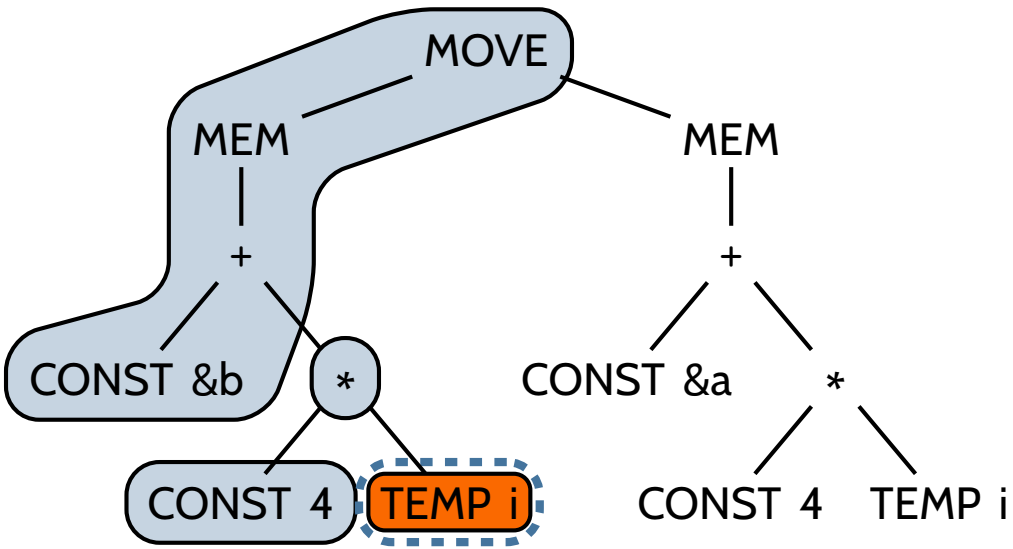
Running maximum munch on our IR tree



Assembly code:

Action: find largest matches

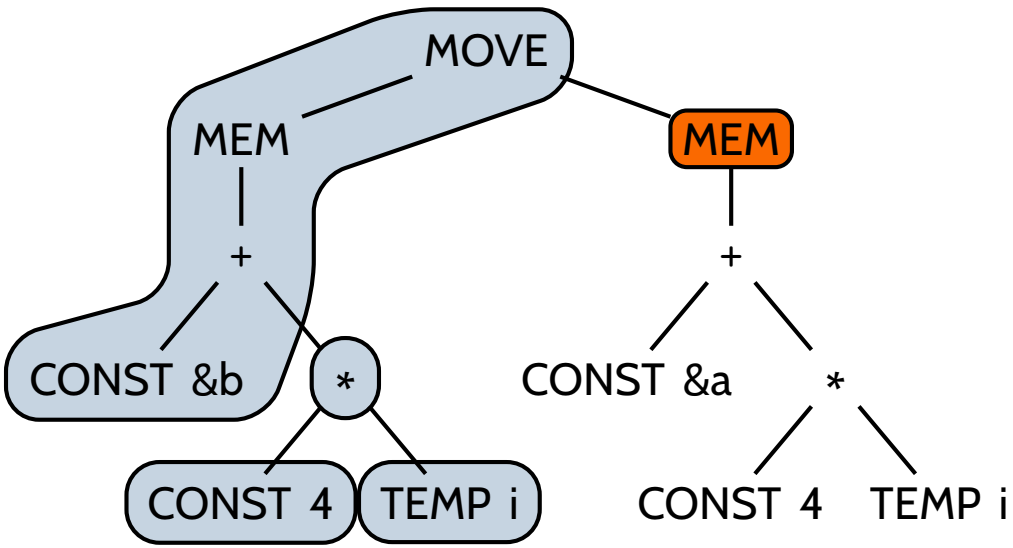
Running maximum munch on our IR tree



Assembly code:

Action: find largest matches

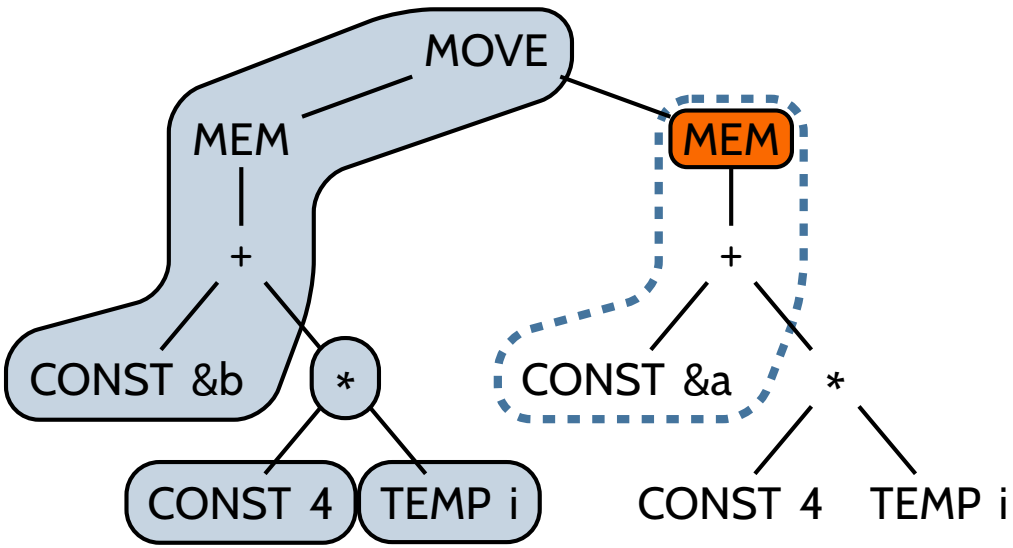
Running maximum munch on our IR tree



Assembly code:

Action: find largest matches

Running maximum munch on our IR tree



Assembly code:

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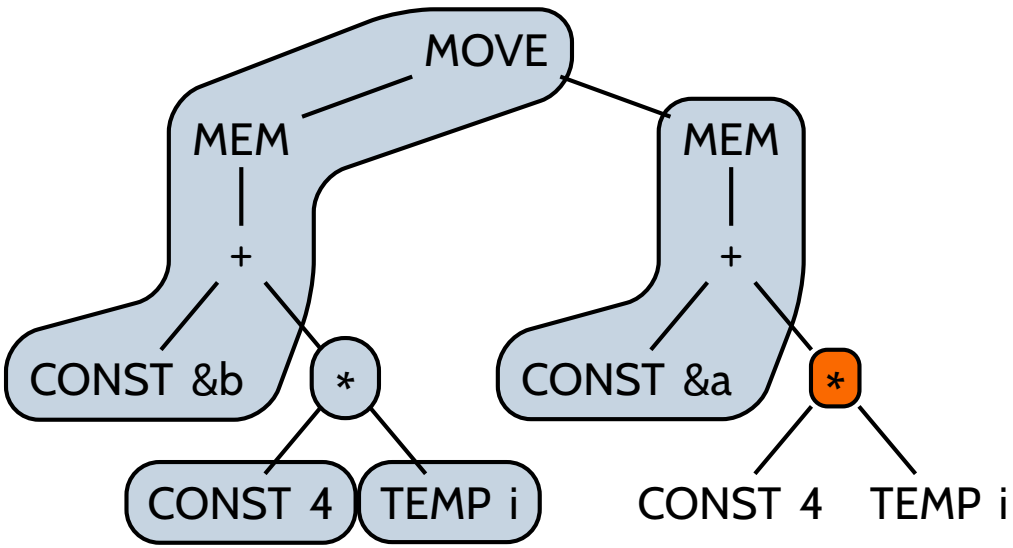
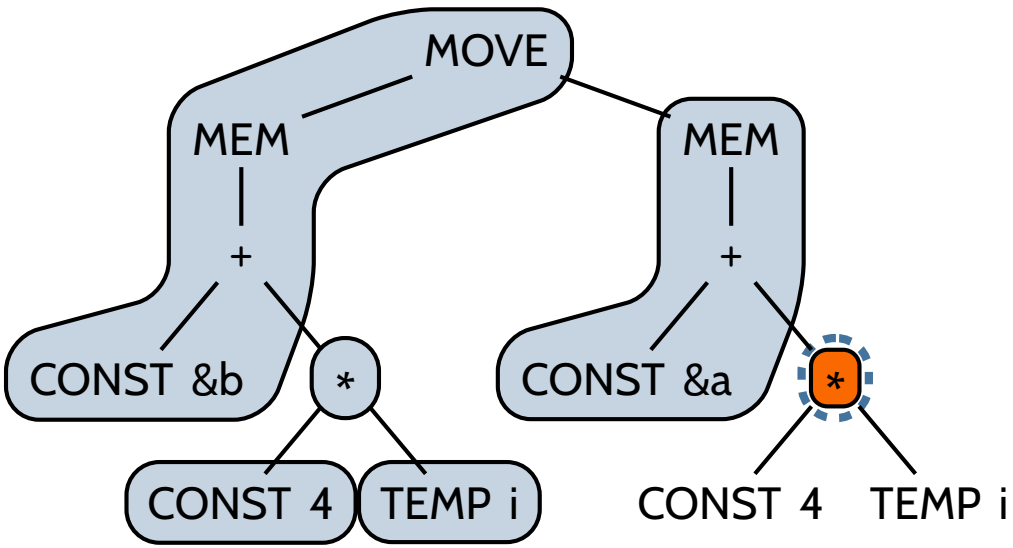
Running maximum munch on our IR tree



Assembly code:

Action: find largest matches

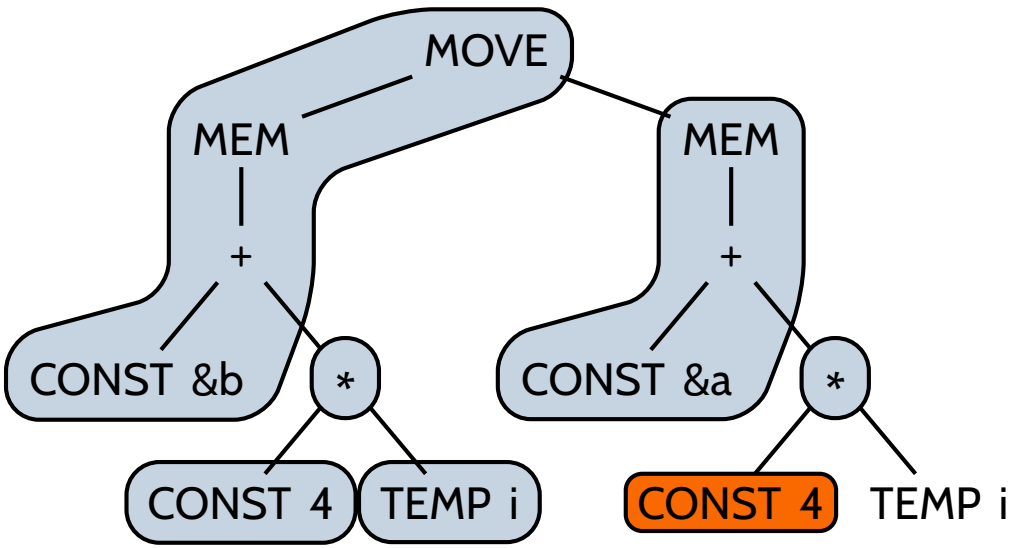
Running maximum munch on our IR tree



Assembly code:

Action: find largest matches

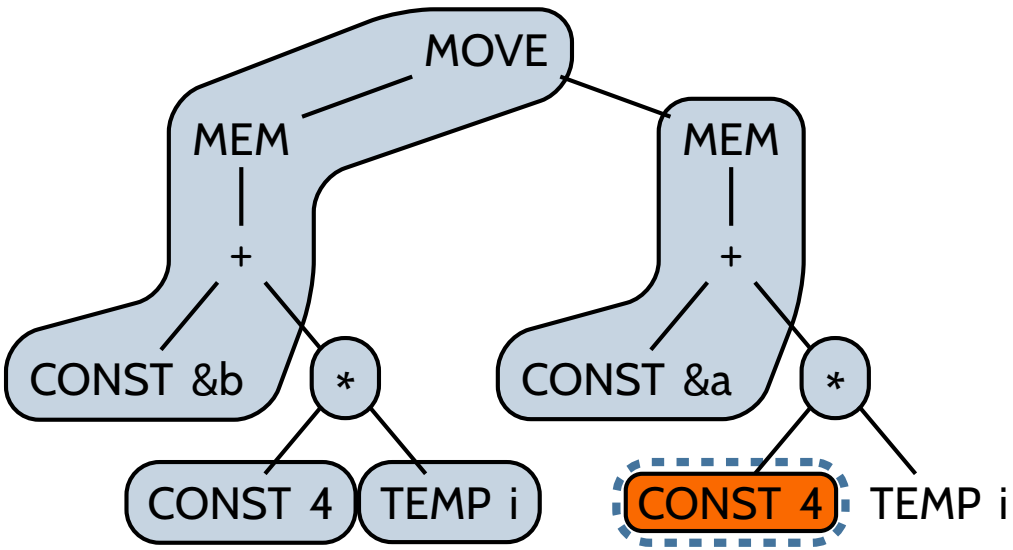
Running maximum munch on our IR tree



Assembly code:

Action: find largest matches

Running maximum munch on our IR tree



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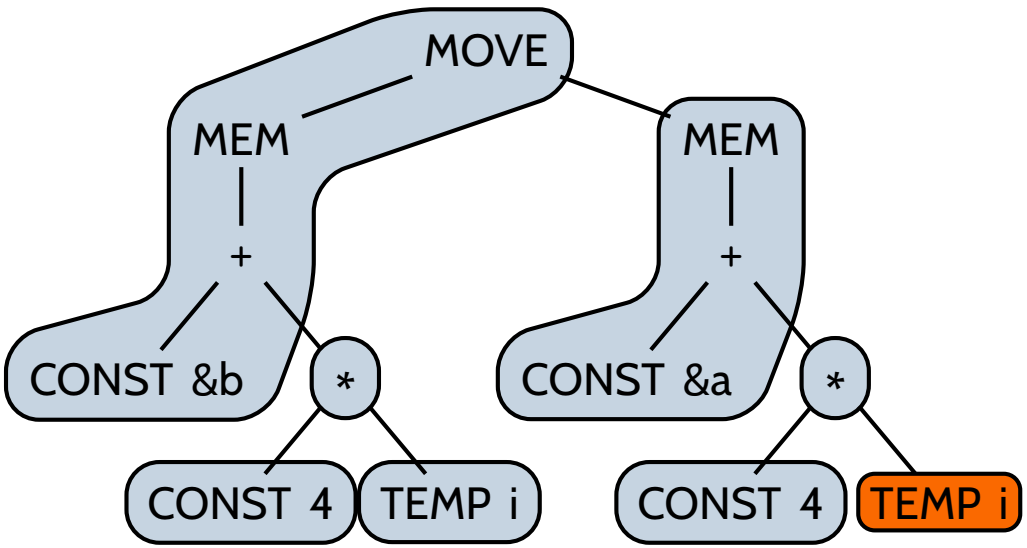
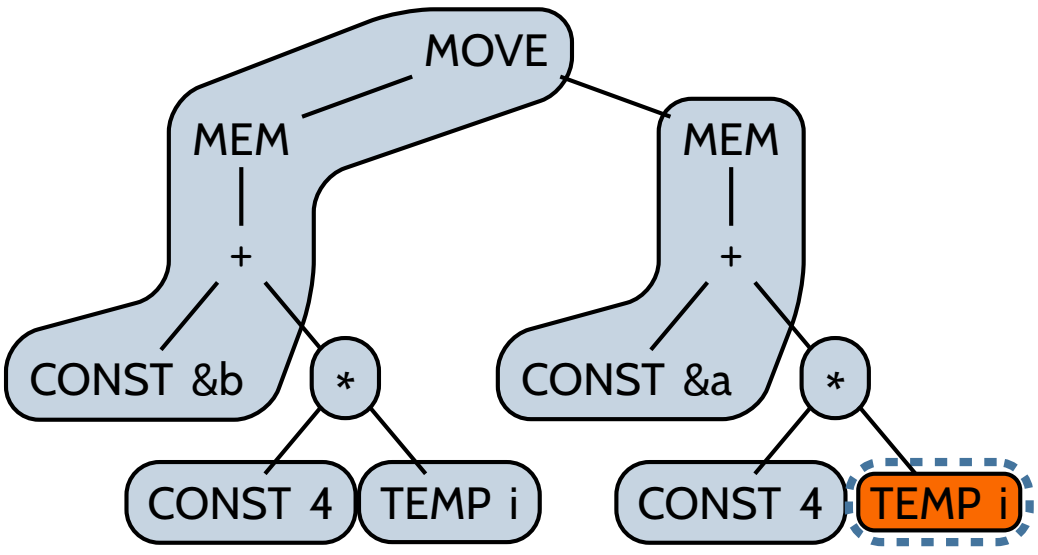
Running maximum munch on our IR tree



Assembly code:

Action: find largest matches

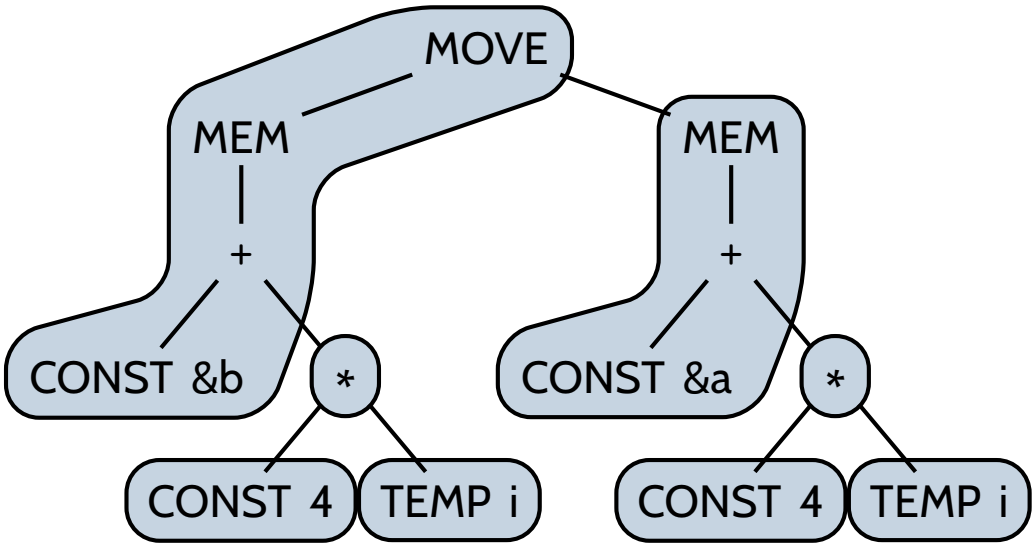
Running maximum munch on our IR tree



Assembly code:

Action: find largest matches

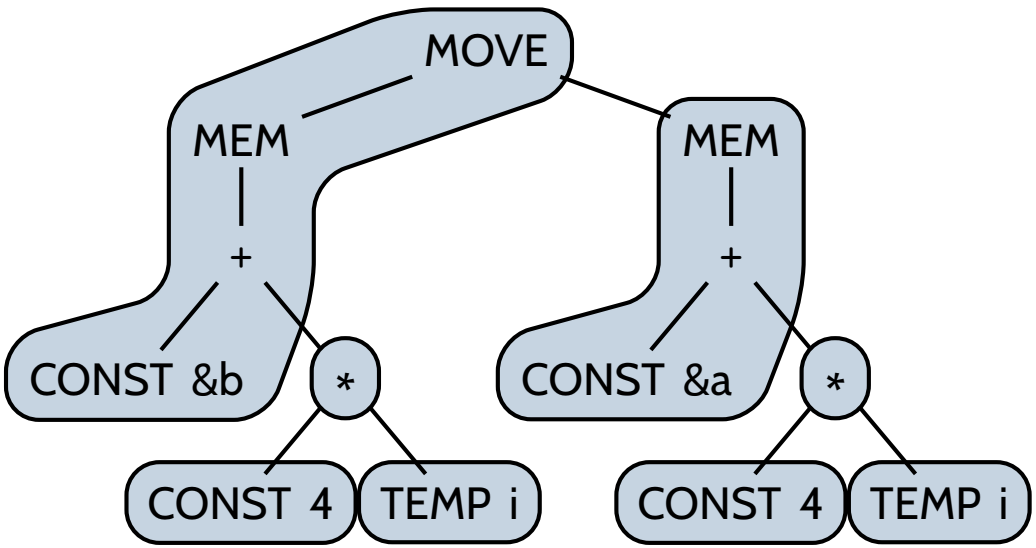
Running maximum munch on our IR tree



Assembly code:

Action: done finding matches

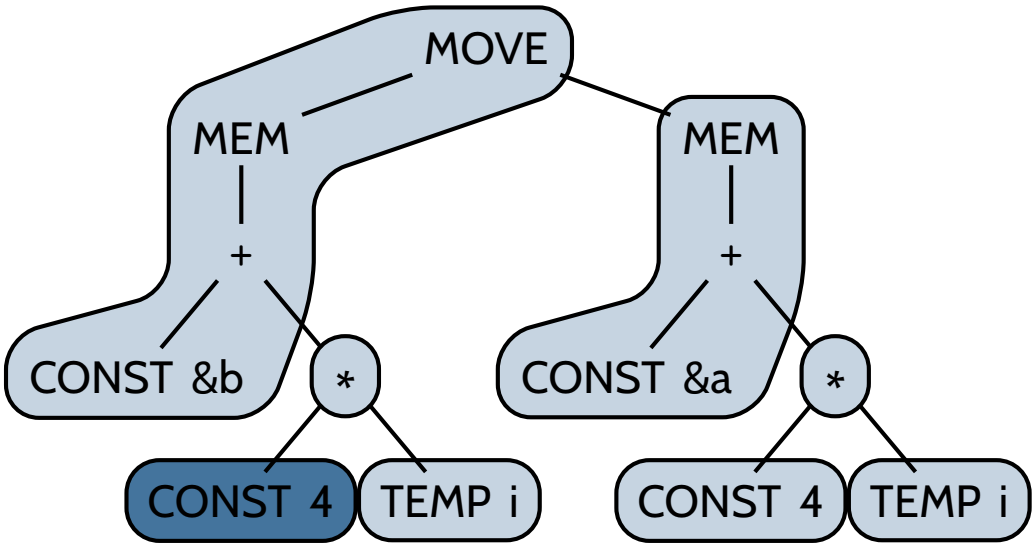
Running maximum munch on our IR tree



Assembly code:

Action: emit assembly instructions

Running maximum munch on our IR tree

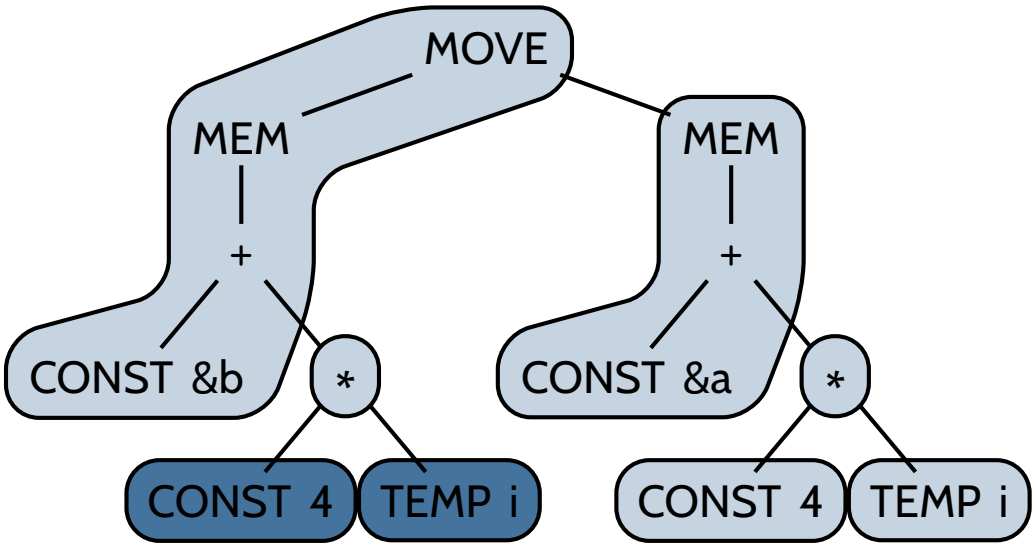


Assembly code:

ADDI $t_0 \leftarrow r_0 + \#4$

Action: emit assembly instructions

Running maximum munch on our IR tree

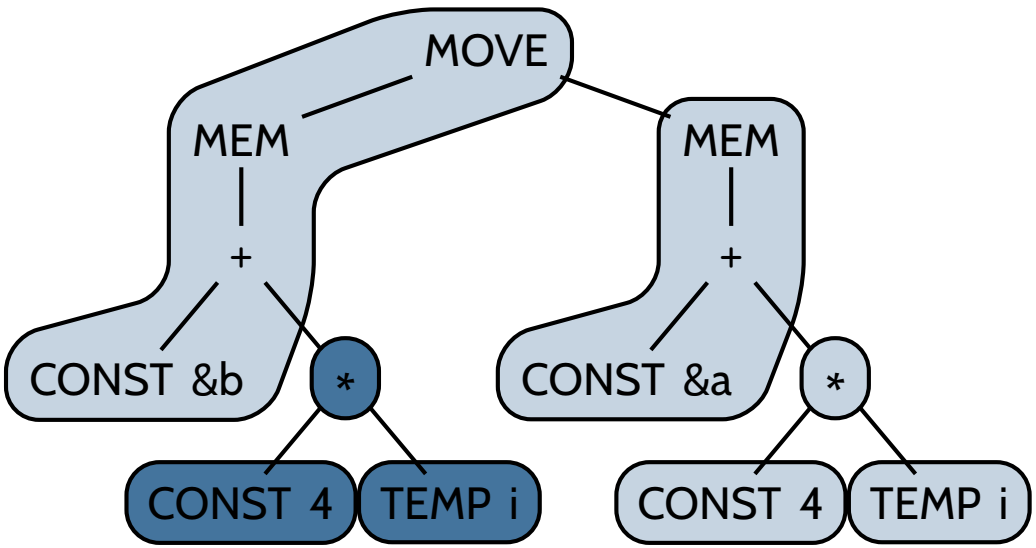


Assembly code:

ADDI $t_0 \leftarrow r_0 + \#4$

Action: emit assembly instructions

Running maximum munch on our IR tree

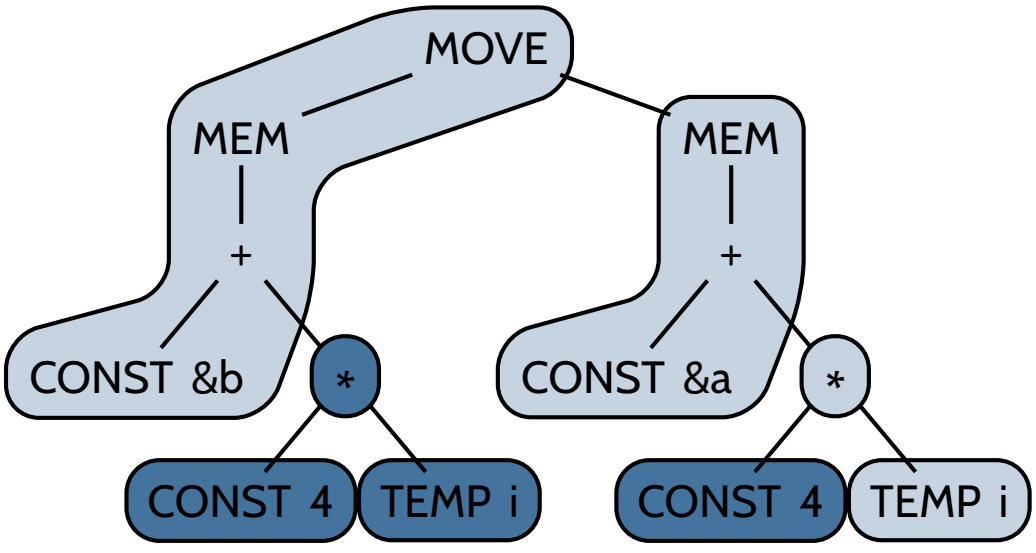


Assembly code:

ADDI $t_0 \leftarrow r_0 + \#4$
MUL $t_1 \leftarrow t_0 * t_i$

Action: emit assembly instructions

Running maximum munch on our IR tree

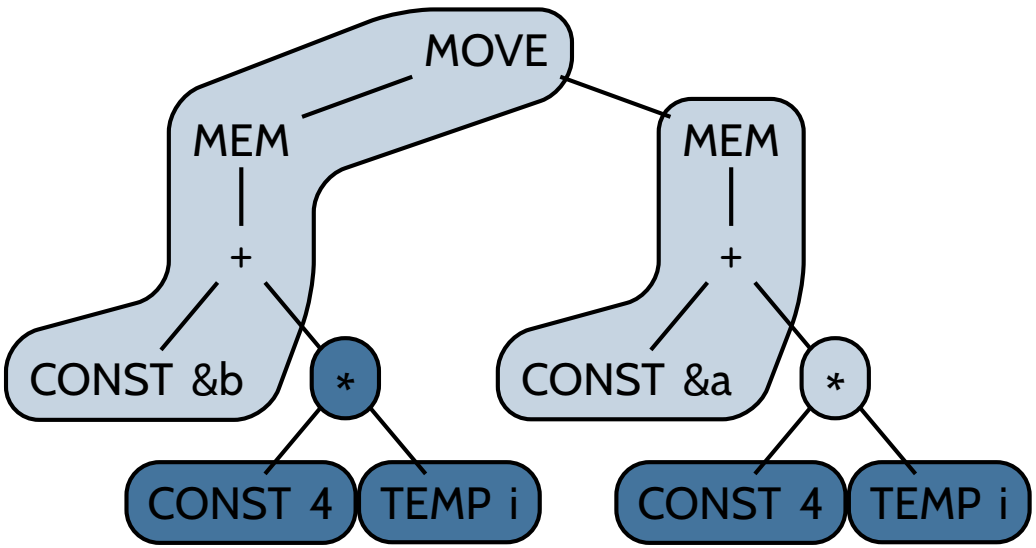


Assembly code:

ADDI $t_0 \leftarrow r_0 + \#4$
MUL $t_1 \leftarrow t_0 * t_i$
ADDI $t_2 \leftarrow r_0 + \#4$

Action: emit assembly instructions

Running maximum munch on our IR tree

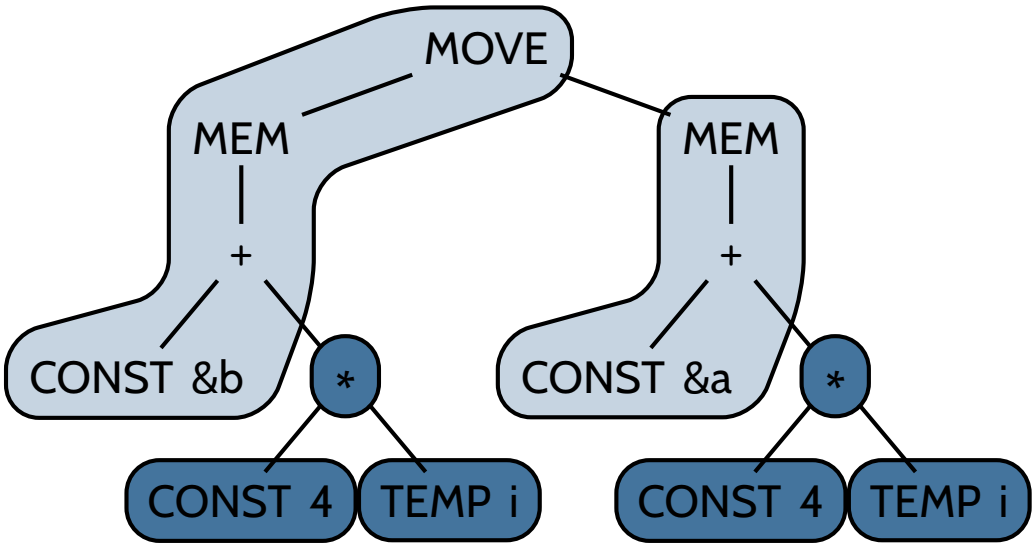


Assembly code:

ADDI $t_0 \leftarrow r_0 + \#4$
MUL $t_1 \leftarrow t_0 * t_i$
ADDI $t_2 \leftarrow r_0 + \#4$

Action: emit assembly instructions

Running maximum munch on our IR tree

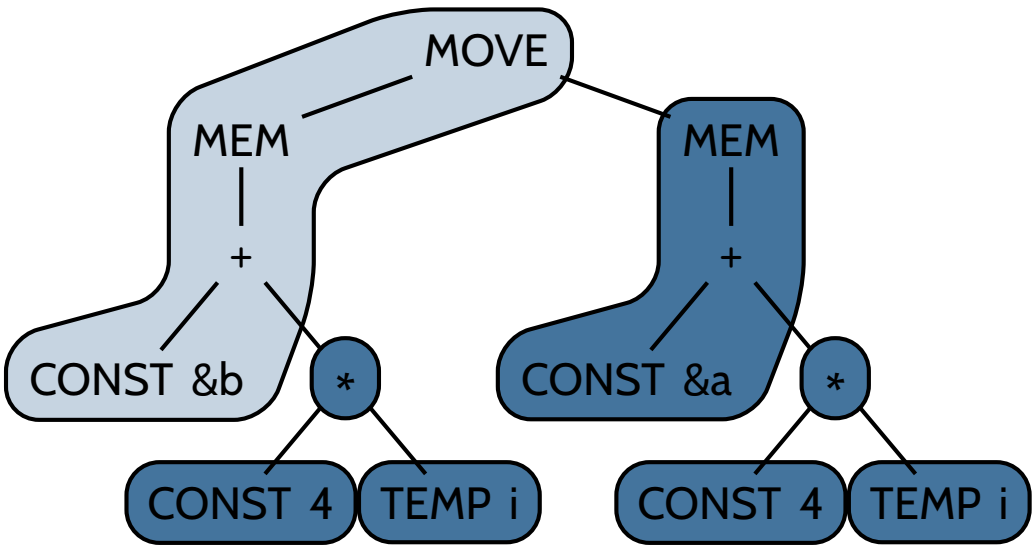


Assembly code:

```
ADDI    t0 ← r0 + #4
MUL     t1 ← t0 * ti
ADDI    t2 ← r0 + #4
MUL     t3 ← t2 * ti
```

Action: emit assembly instructions

Running maximum munch on our IR tree

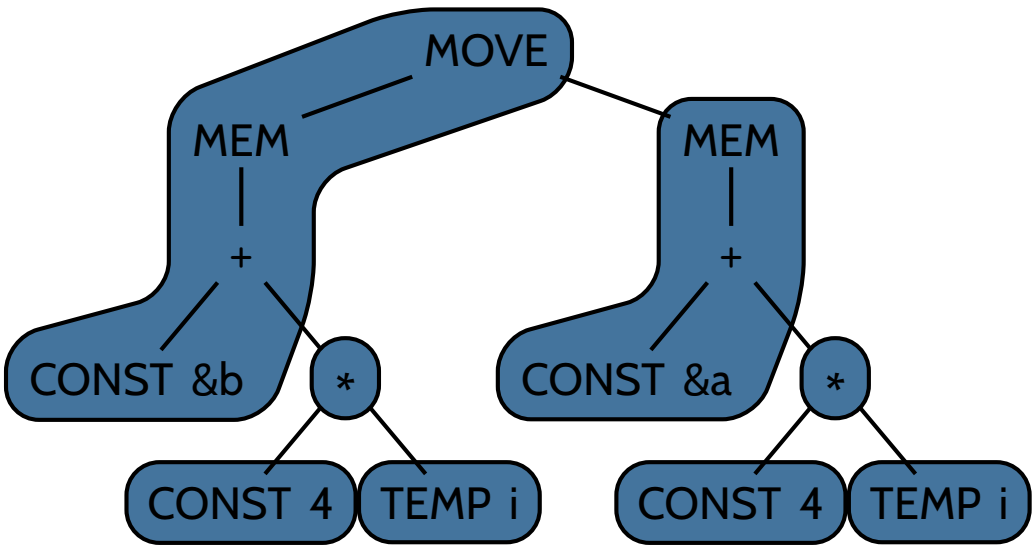


Assembly code:

ADDI $t_0 \leftarrow r_0 + \#4$
MUL $t_1 \leftarrow t_0 * t_i$
ADDI $t_2 \leftarrow r_0 + \#4$
MUL $t_3 \leftarrow t_2 * t_i$
LOAD $t_4 \leftarrow M[t_3 + \#\&a]$

Action: emit assembly instructions

Running maximum munch on our IR tree



Assembly code:

```
ADDI    t0 ← r0 + #4
MUL     t1 ← t0 * ti
ADDI    t2 ← r0 + #4
MUL     t3 ← t2 * ti
LOAD    t4 ← M[t3 + #&a]
STORE   M[t1 + #&b] ← t4
```

Action: done

Quality of emitted assembly code

		Costs:
ADDI	$t_0 \leftarrow r_0 + \#4$	1
MUL	$t_1 \leftarrow t_0 * t_i$	2
ADDI	$t_2 \leftarrow r_0 + \#4$	1
MUL	$t_3 \leftarrow t_2 * t_i$	2
LOAD	$t_4 \leftarrow M[t_3 + \#\&a]$	10
STORE	$M[t_1 + \#\&b] \leftarrow t_4$	10

$$\sum \text{cost} = 26$$

3rd approach:

Tree Parsing

Fundamental idea

- Derive **tree grammar** from tile set:
  - Each tile yields a production
- **Generate** LR parser from tree grammar
- To find tile matches and optimal tiling:
  1. Transform IR tree into an **IR string**
  2. Run LR parser on IR string
- To emit assembly code:
  - When performing a reduction:
    - Emit corresponding instruction

Transforming trees into strings

- **Polish notation:**
  - Operator is placed *in front* of arguments
  - Parentheses superfluous if all operators have fixed **arity** (number of arguments)

Transforming trees into strings

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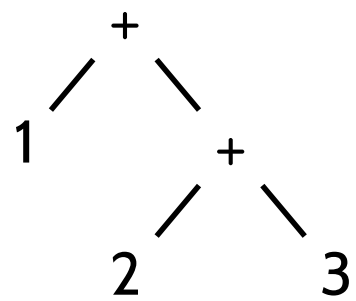

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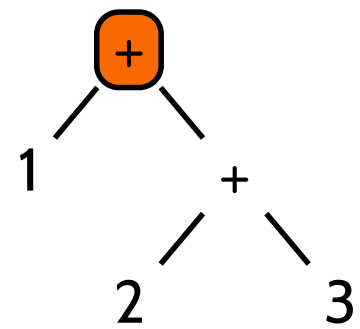

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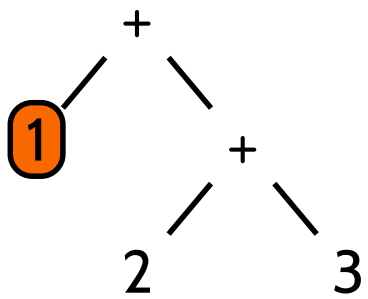

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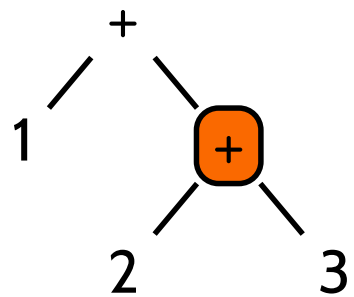

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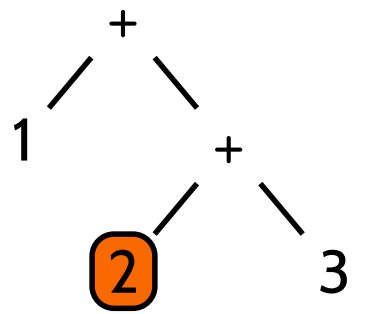

Transforming trees into strings

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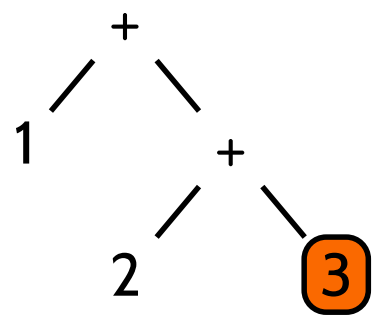

+ 1 + 2 3

Transforming trees into strings

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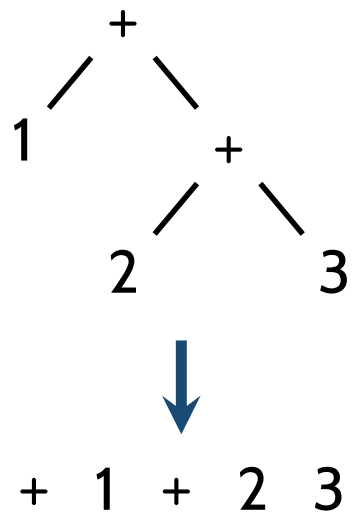

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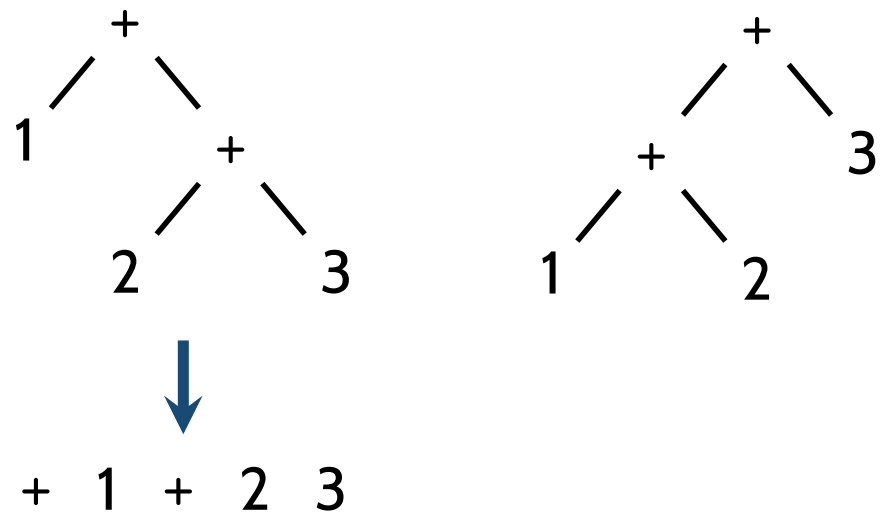
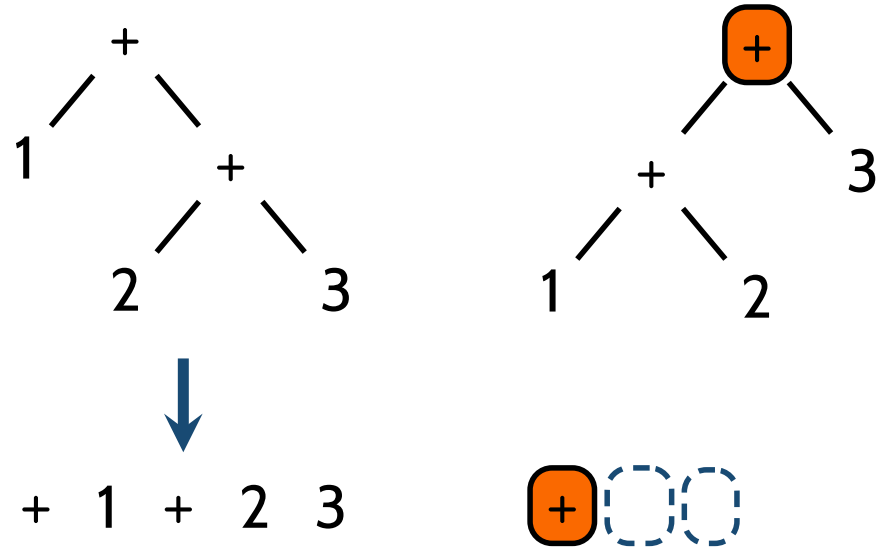


Transforming trees into strings

- **Polish notation:**
  - Operator is placed *in front* of arguments
  - Parentheses superfluous if all operators have fixed **arity** (number of arguments)



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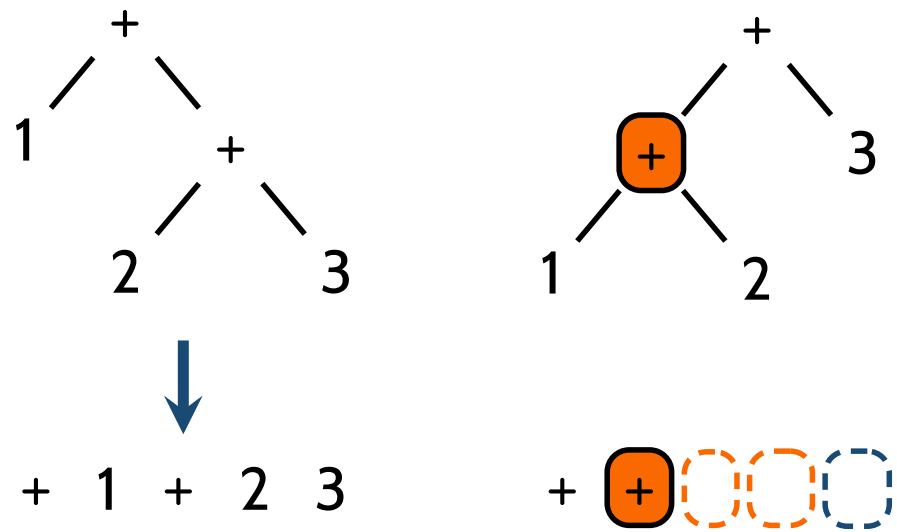

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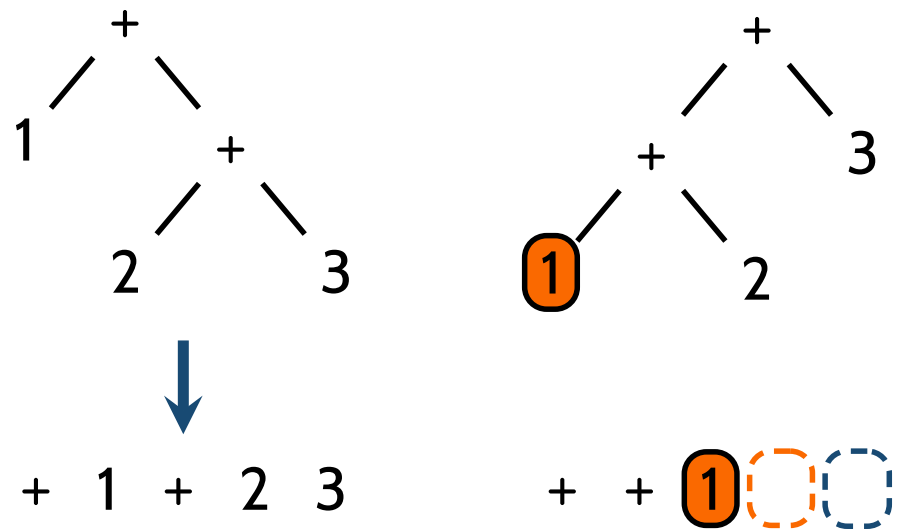

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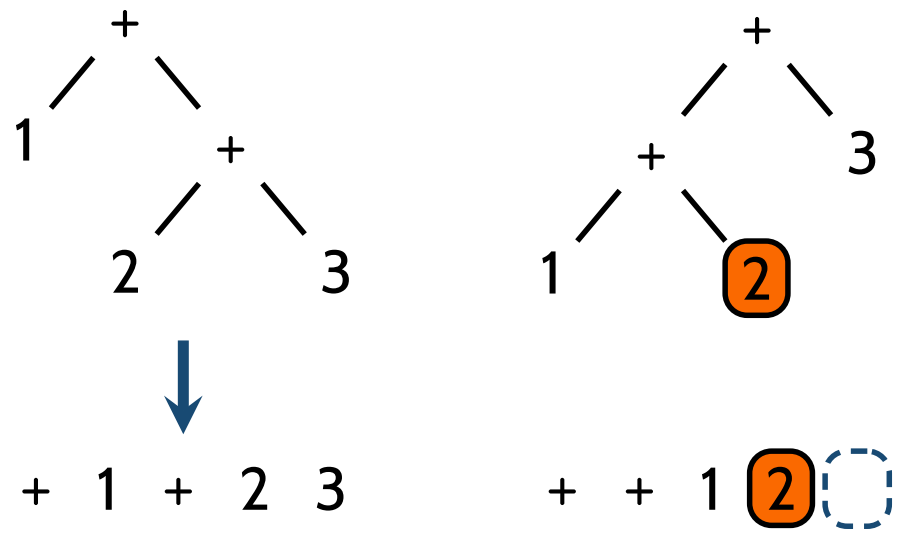

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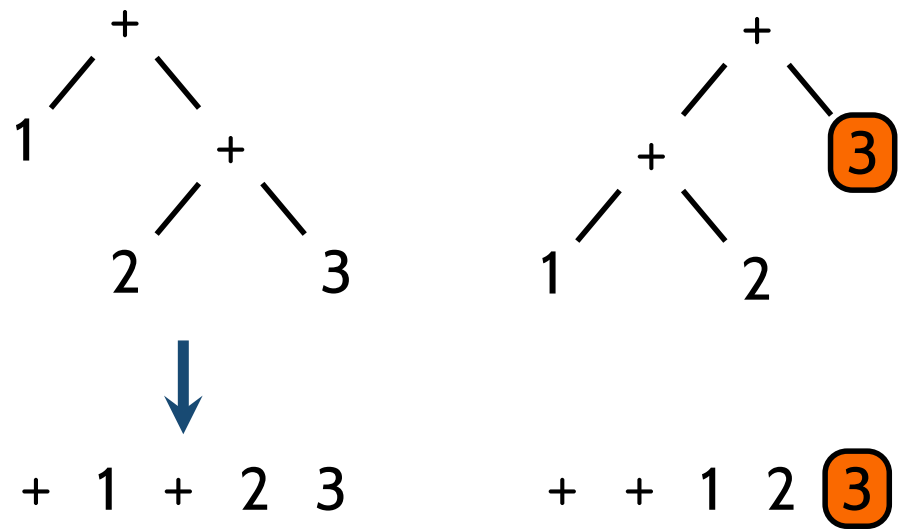

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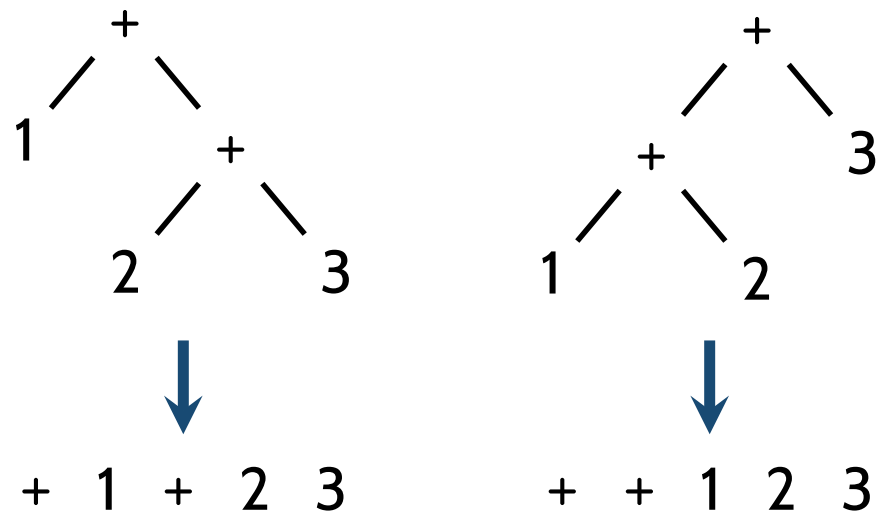

Transforming trees into strings

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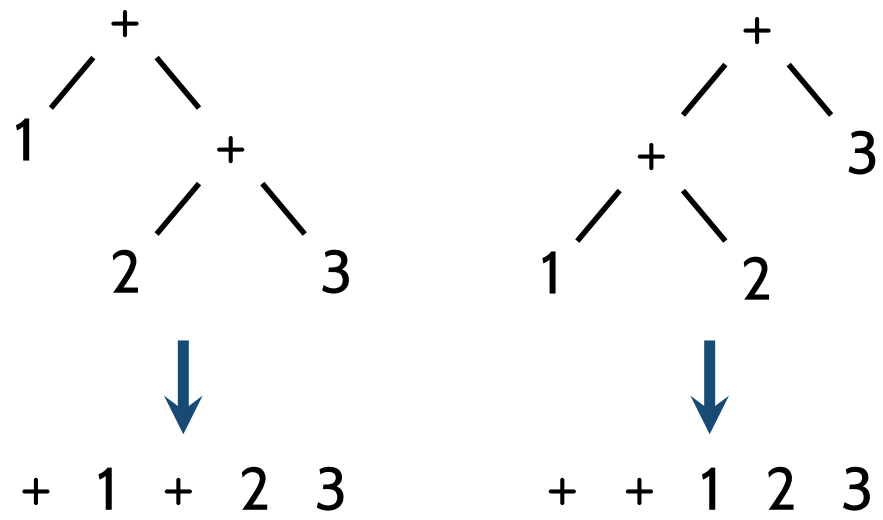
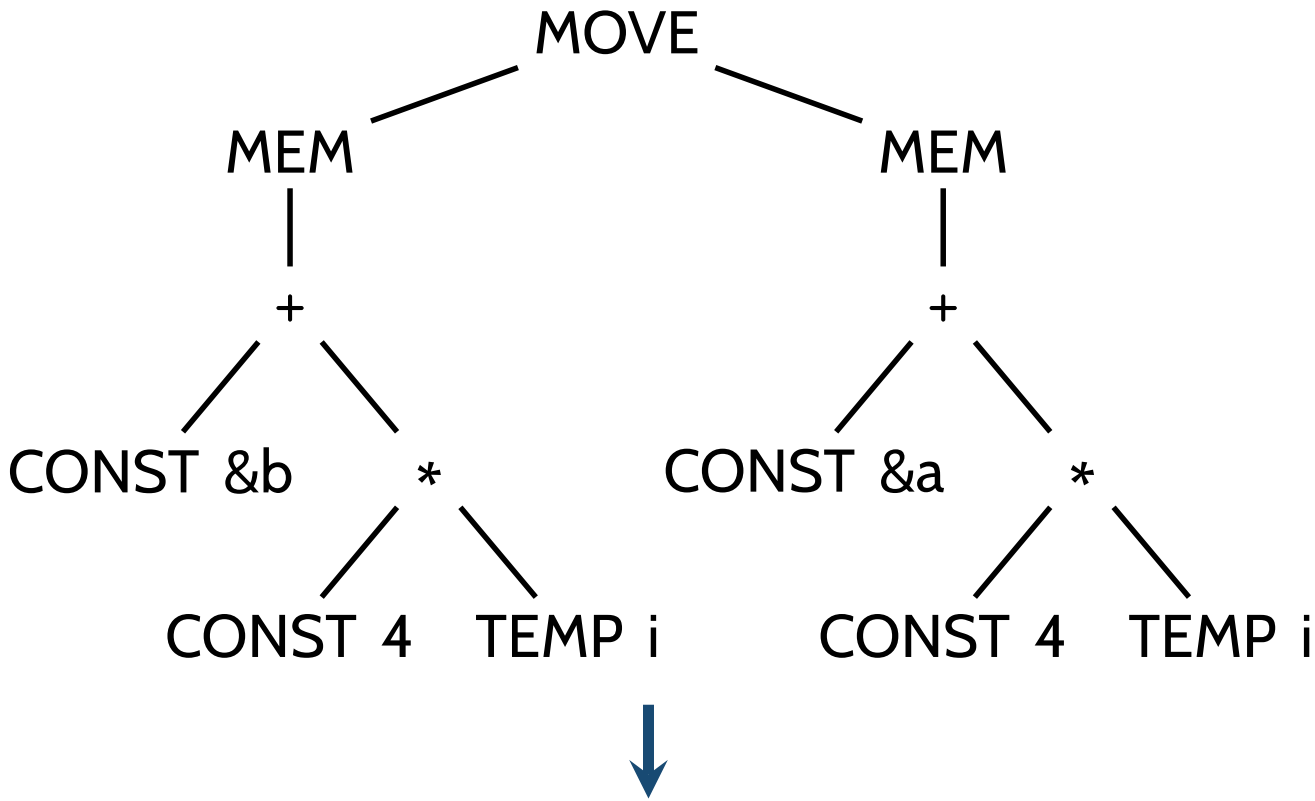


- Equivalent to depth-first, left-to-right tree traversal

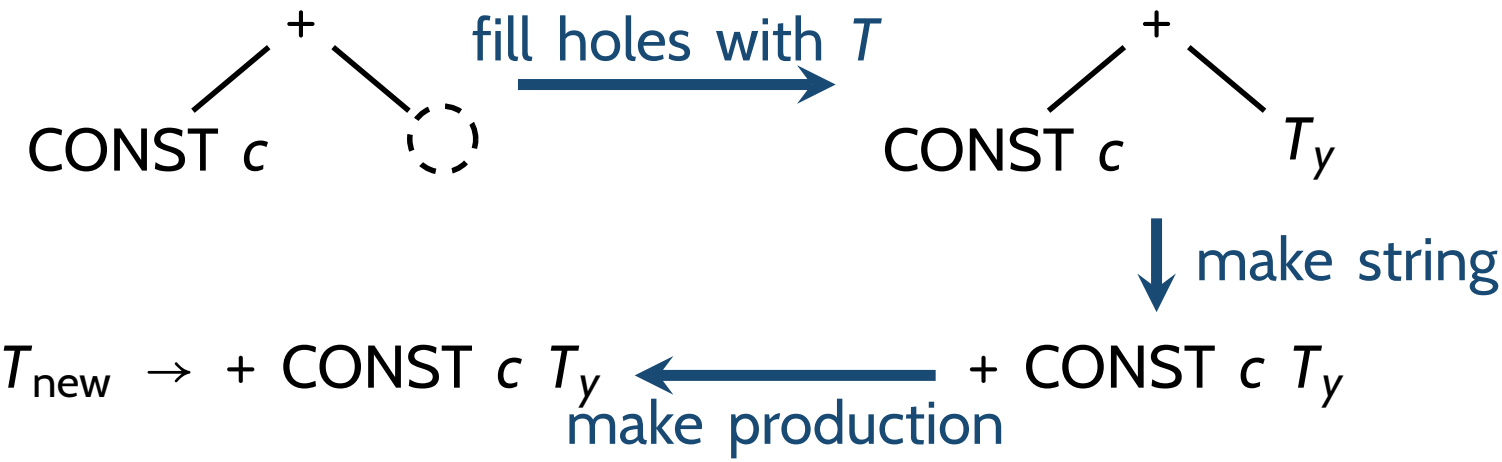
Transforming our IR tree into an IR string



MOVE MEM + CONST &b * CONST 4 TEMP i
MEM + CONST &a * CONST 4 TEMP i

Deriving the tree grammar

- Introduce nonterminal T_t to represent temporaries
  - Subscript refers to some specific temporary t
- Introduce start symbol:
  - $S \rightarrow T$ \$
- For each tile:



- In relation to expansion macros:
  - T_y corresponds to "t_y = getResultOf(E_{rhs})"
  - T_{new} corresponds to "t_{new} = getNewTemp() . . . setResultIsIn(t_{new})"

Tree grammar for *Jouette*

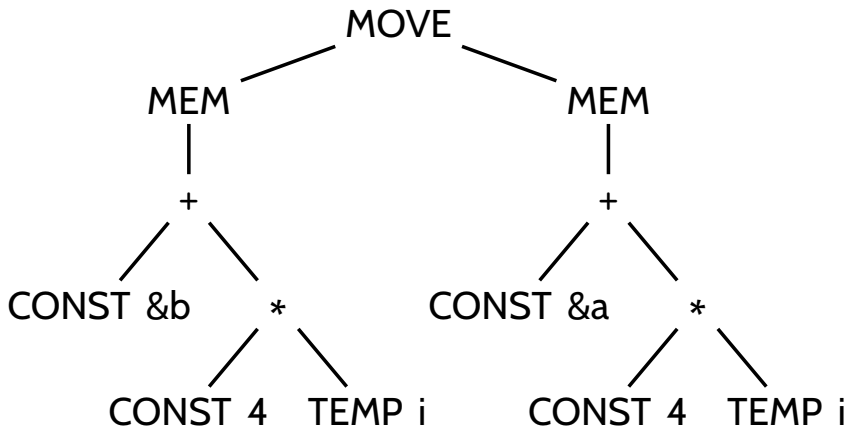
Instruction		Productions
–		$T_t \rightarrow \text{TEMP } t$
ADD	$t_{new} \leftarrow t_y + t_z$	$T_{new} \rightarrow + \ T_y \ T_z$
MUL	$t_{new} \leftarrow t_y * t_z$	$T_{new} \rightarrow * \ T_y \ T_z$
ADDI	$t_{new} \leftarrow t_y + \#c$	$T_{new} \rightarrow + \ T_y \ \text{CONST } c$ $T_{new} \rightarrow + \ \text{CONST } c \ T_y$ $T_{new} \rightarrow \text{CONST } c$
LOAD	$t_{new} \leftarrow M[t_y + \#c]$	$T_{new} \rightarrow \text{MEM} + \ T_y \ \text{CONST } c$ $T_{new} \rightarrow \text{MEM} + \ \text{CONST } c \ T_y$ $T_{new} \rightarrow \text{MEM CONST } c$ $T_{new} \rightarrow \text{MEM } T_y$
STORE	$M[t_x + \#c] \leftarrow t_y$	$T \rightarrow \text{MOVE MEM} + \ \text{CONST } c \ T_x \ T_y$ $T \rightarrow \text{MOVE MEM} + \ T_x \ \text{CONST } c \ T_y$ $T \rightarrow \text{MOVE MEM CONST } c \ T_y$ $T \rightarrow \text{MOVE MEM } T_x \ T_y$
MOVEM	$M[t_x] \leftarrow M[t_y]$	$T \rightarrow \text{MOVE MEM } T_x \ \text{MEM } T_y$

Tree grammars derived from assembly instructions often highly ambiguous

- Resolving shift-reduce conflicts:
  - Always shift
- Resolving reduce-reduce conflicts:
  - Choose longest production

Heuristic above equivalent to maximum munch

Running tree parsing on our IR tree



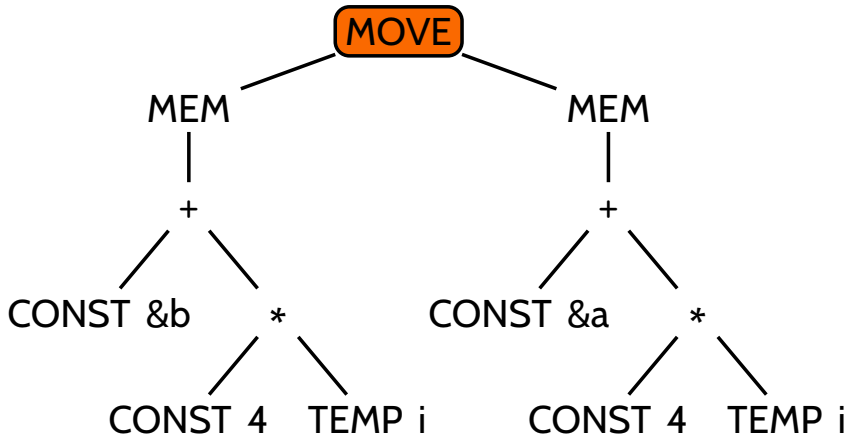
Assembly code:

MOVE MEM + CONST &b * CONST 4 TEMP i
MEM + CONST &a * CONST 4 TEMP i $

Action:

Stack:

Running tree parsing on our IR tree



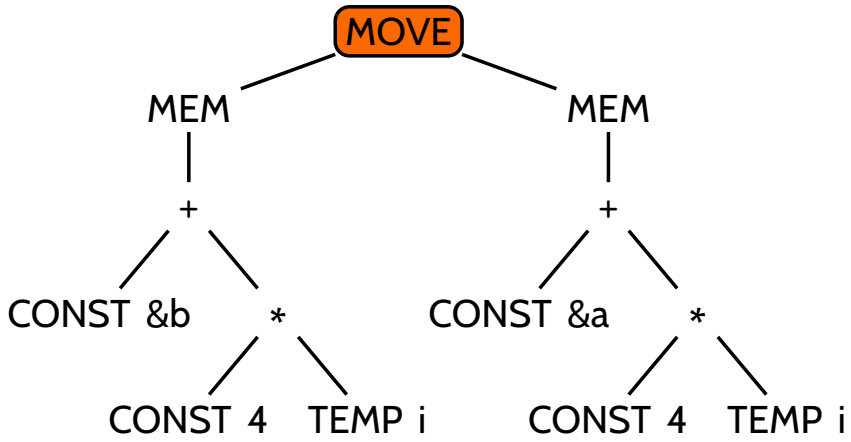
Assembly code:

MOVE MEM + CONST &b * CONST 4 TEMP i
MEM + CONST &a * CONST 4 TEMP i $

Action: move to next symbol

Stack:

Running tree parsing on our IR tree



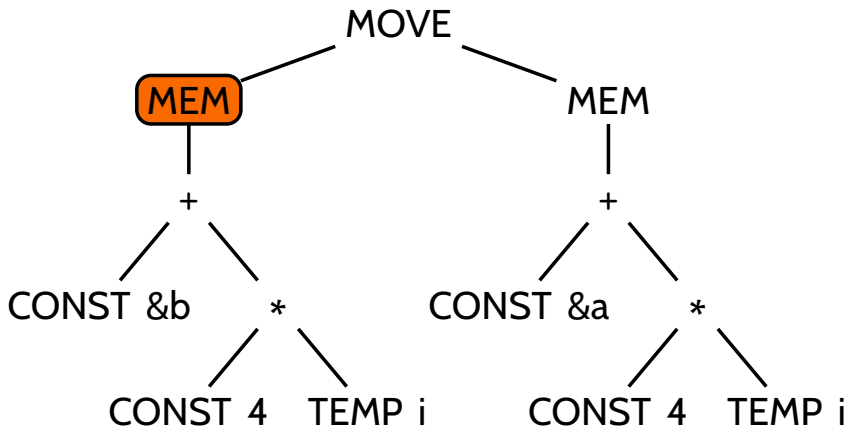
Assembly code:

MOVE MEM + CONST &b * CONST 4 TEMP i
MEM + CONST &a * CONST 4 TEMP i $

Action: shift

Stack: MOVE

Running tree parsing on our IR tree



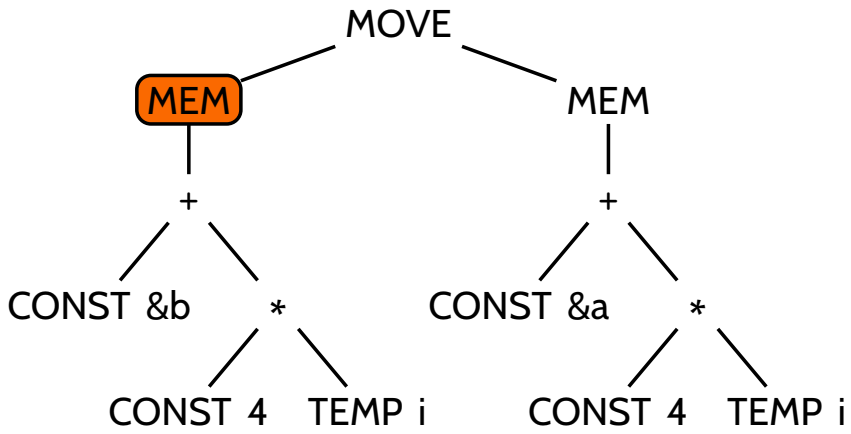
Assembly code:

MOVE MEM + CONST &b * CONST 4 TEMP i
MEM + CONST &a * CONST 4 TEMP i $

Action: move to next symbol

Stack: MOVE

Running tree parsing on our IR tree



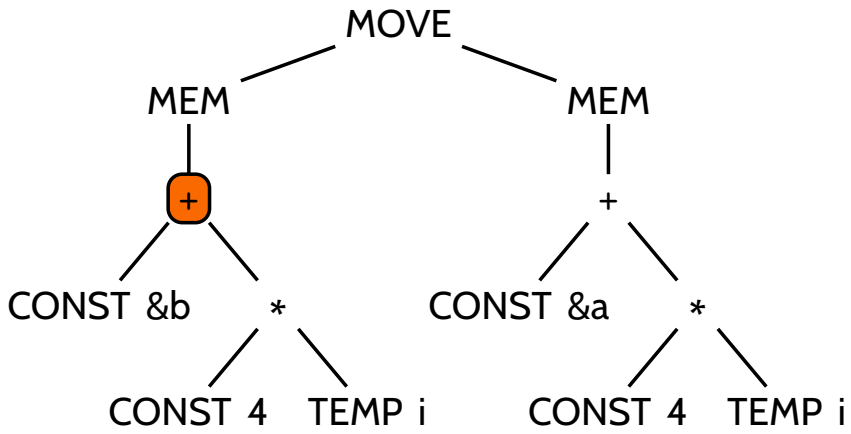
Assembly code:

MOVE MEM + CONST &b * CONST 4 TEMP i
MEM + CONST &a * CONST 4 TEMP i $

Action: shift

Stack: MOVE MEM

Running tree parsing on our IR tree



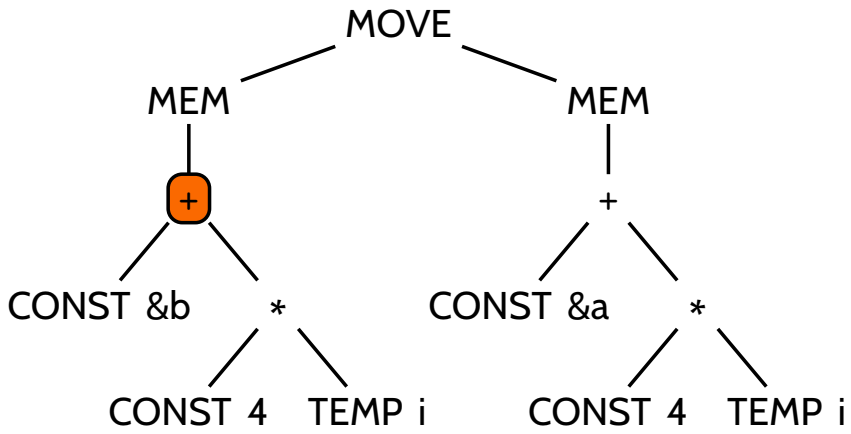
Assembly code:

MOVE MEM + CONST &b * CONST 4 TEMP i
MEM + CONST &a * CONST 4 TEMP i $

Action: move to next symbol

Stack: MOVE MEM

Running tree parsing on our IR tree



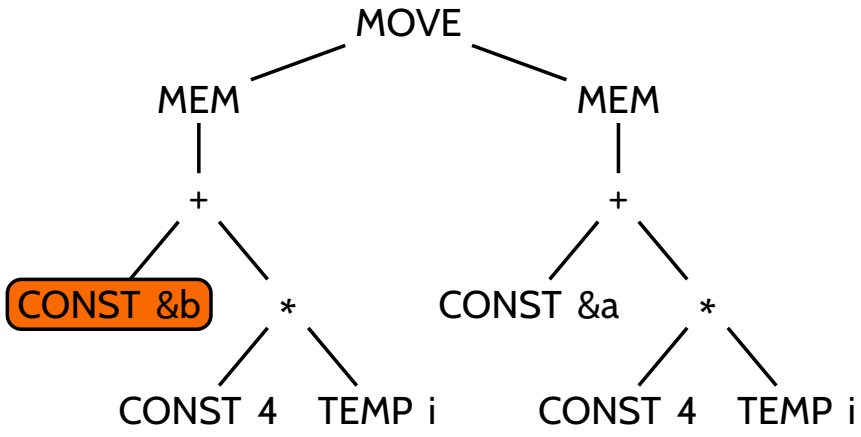
Assembly code:

MOVE MEM + CONST &b * CONST 4 TEMP i
MEM + CONST &a * CONST 4 TEMP i $

Action: shift

Stack: MOVE MEM +

Running tree parsing on our IR tree



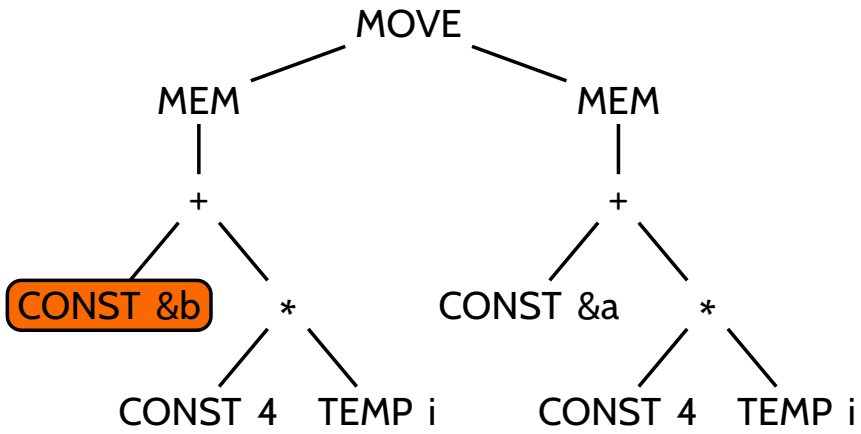
Assembly code:

MOVE MEM + CONST &b * CONST 4 TEMP i
MEM + CONST &a * CONST 4 TEMP i $

Action: move to next symbol

Stack: MOVE MEM +

Running tree parsing on our IR tree



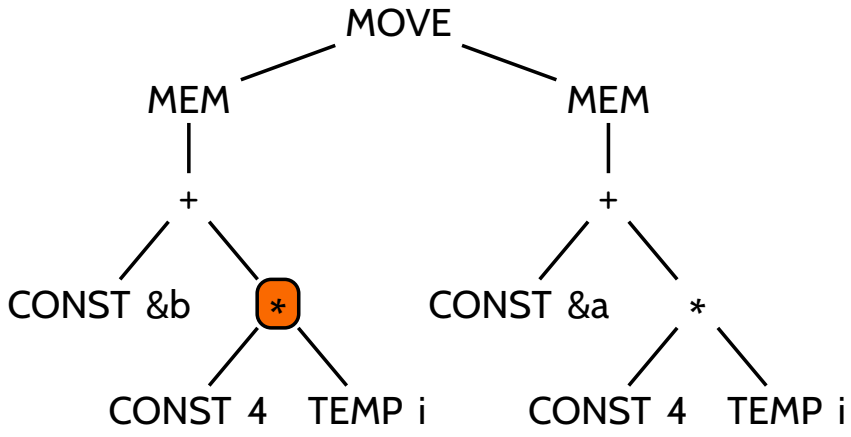
Assembly code:

MOVE MEM + CONST &b * CONST 4 TEMP i
MEM + CONST &a * CONST 4 TEMP i $

Action: shift

Stack: MOVE MEM + CONST &b

Running tree parsing on our IR tree



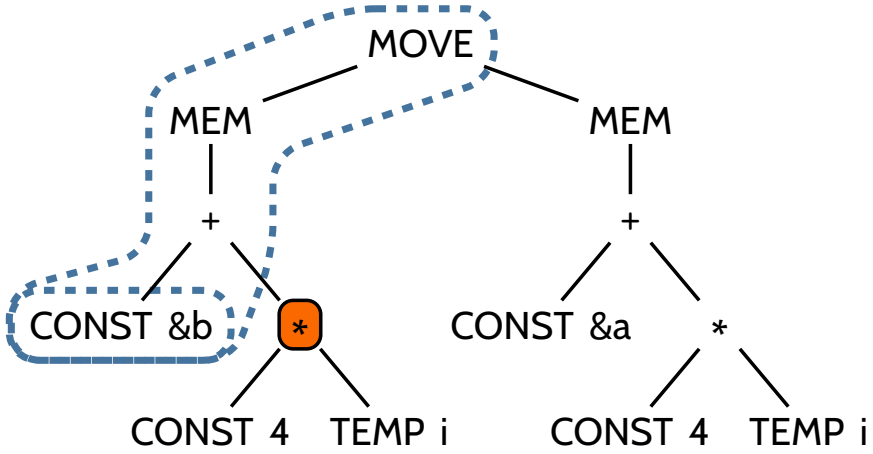
Assembly code:

MOVE MEM + CONST &b * CONST 4 TEMP i
MEM + CONST &a * CONST 4 TEMP i $

Action: move to next symbol

Stack: MOVE MEM + CONST &b

Running tree parsing on our IR tree



Assembly code:

MOVE MEM + CONST &b * CONST 4 TEMP i
MEM + CONST &a * CONST 4 TEMP i $

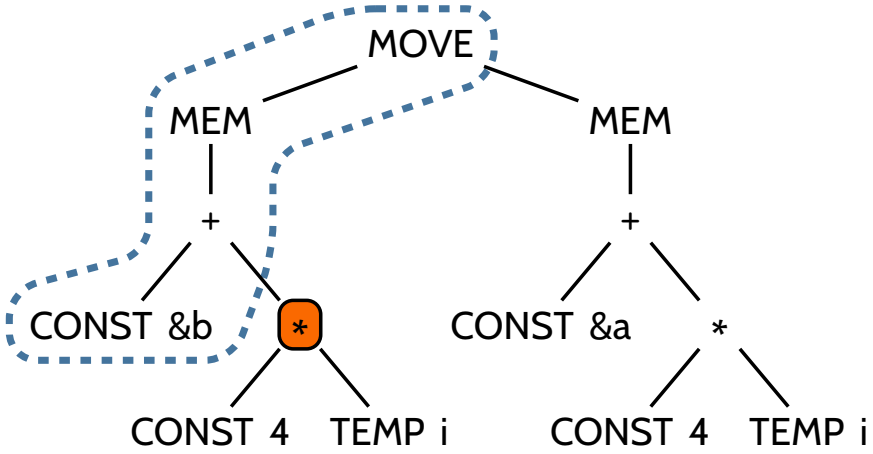
Action: shift-reduce conflict!

Stack: MOVE MEM + CONST &b *

$T_{new} \rightarrow$ CONST c *reducible now*

$T \rightarrow$ MOVE MEM + CONST c T_x T_y *may be reducible later*

Running tree parsing on our IR tree



Assembly code:

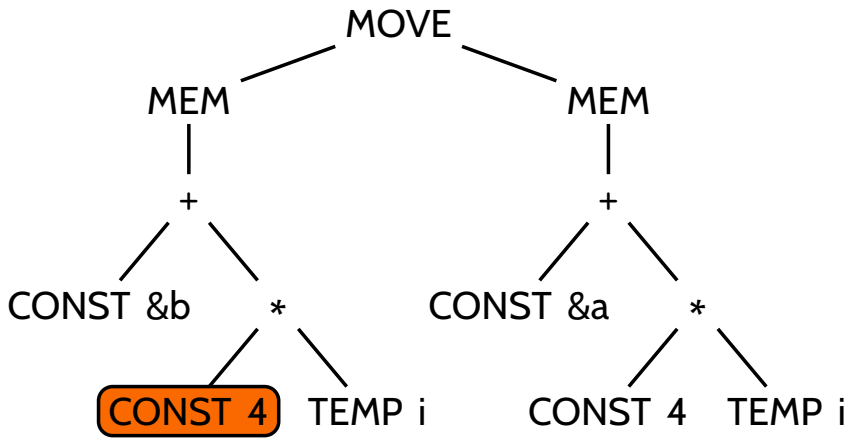
MOVE MEM + CONST &b * CONST 4 TEMP i
MEM + CONST &a * CONST 4 TEMP i $

Action: always shift

Stack: MOVE MEM + CONST &b *

$T \rightarrow$ MOVE MEM + CONST c T_x T_y *hope for later reduction*

Running tree parsing on our IR tree



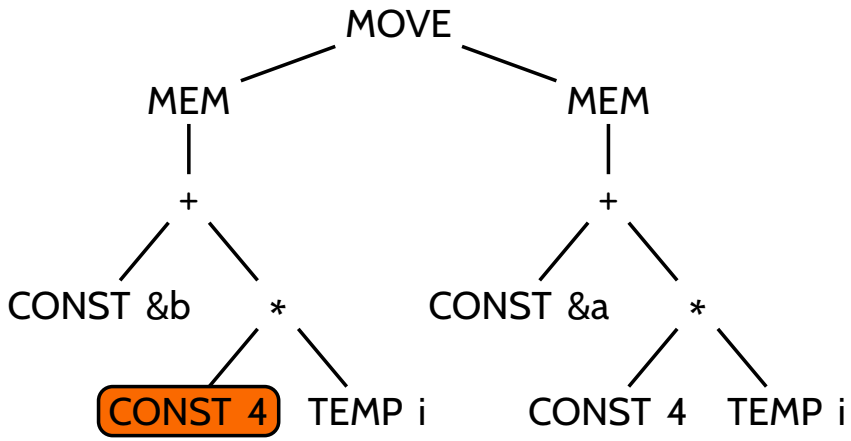
Assembly code:

MOVE MEM + CONST &b * CONST 4 TEMP i
MEM + CONST &a * CONST 4 TEMP i $

Action: move to next symbol

Stack: MOVE MEM + CONST &b *

Running tree parsing on our IR tree



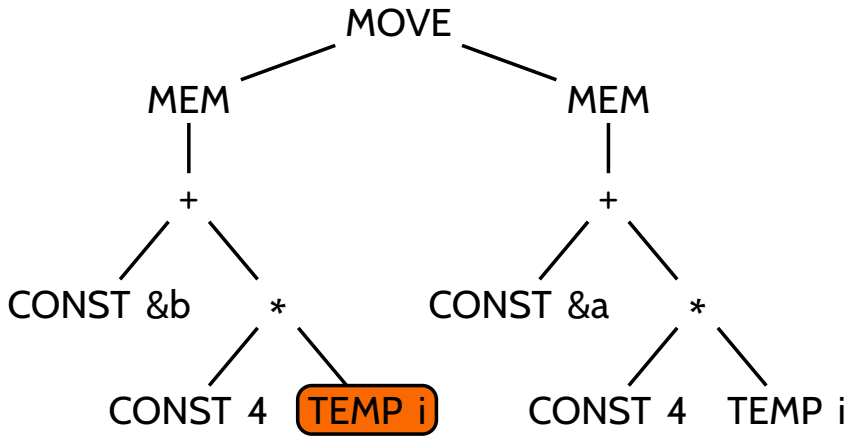
Assembly code:

MOVE MEM + CONST &b * CONST 4 TEMP i
MEM + CONST &a * CONST 4 TEMP i $

Action: shift

Stack: MOVE MEM + CONST &b * CONST 4

Running tree parsing on our IR tree



Assembly code:

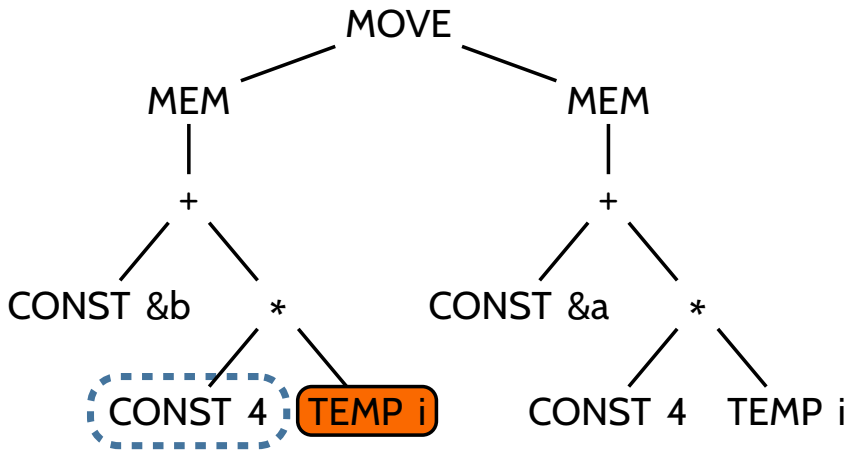
MOVE MEM + CONST &b * CONST 4 TEMP i

MEM + CONST &a * CONST 4 TEMP i $

Action: move to next symbol

Stack: MOVE MEM + CONST &b * CONST 4

Running tree parsing on our IR tree



Assembly code:

MOVE MEM + CONST &b * CONST 4 TEMP i

MEM + CONST &a * CONST 4 TEMP i $

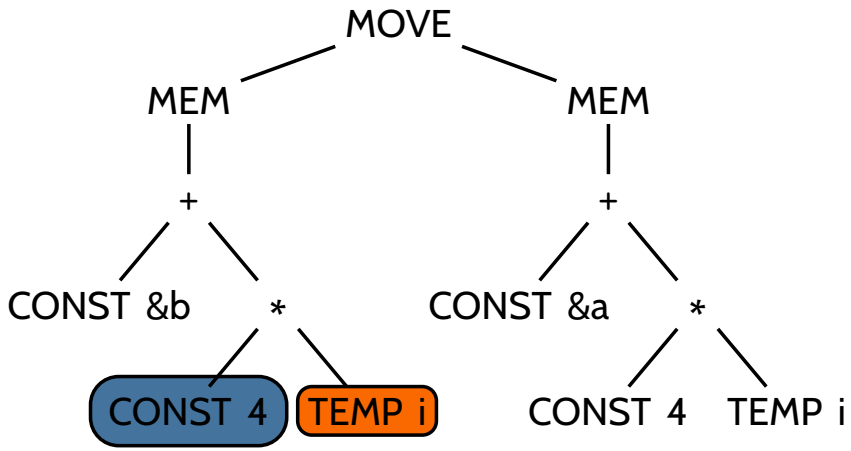
Action: reduce

Stack: MOVE MEM + CONST &b * CONST 4

$T_{new} \rightarrow$ CONST c

due to $T_{new} \rightarrow * \; T_y \; T_z$

Running tree parsing on our IR tree



MOVE MEM + CONST &b * CONST 4 TEMP i

MEM + CONST &a * CONST 4 TEMP i $

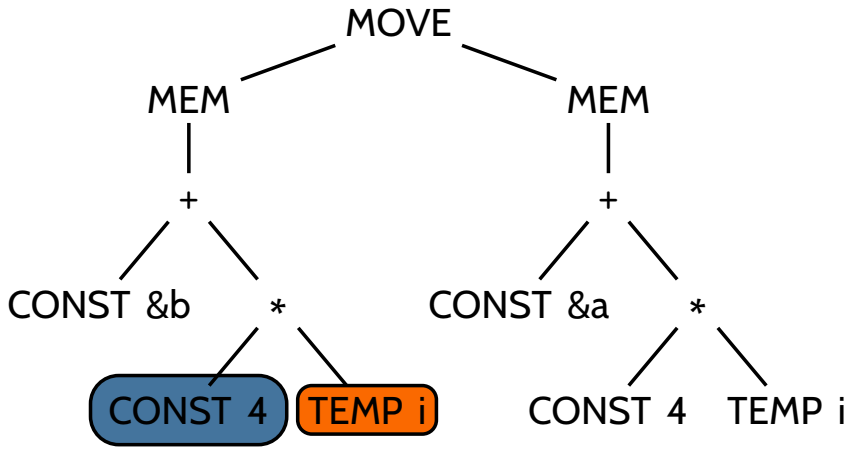
Action:

Stack: MOVE MEM + CONST &b * T_0

Assembly code:

ADDI $t_0 \leftarrow r_0 + \#4$

Running tree parsing on our IR tree



Assembly code:

ADDI $t_0 \leftarrow r_0 + \#4$

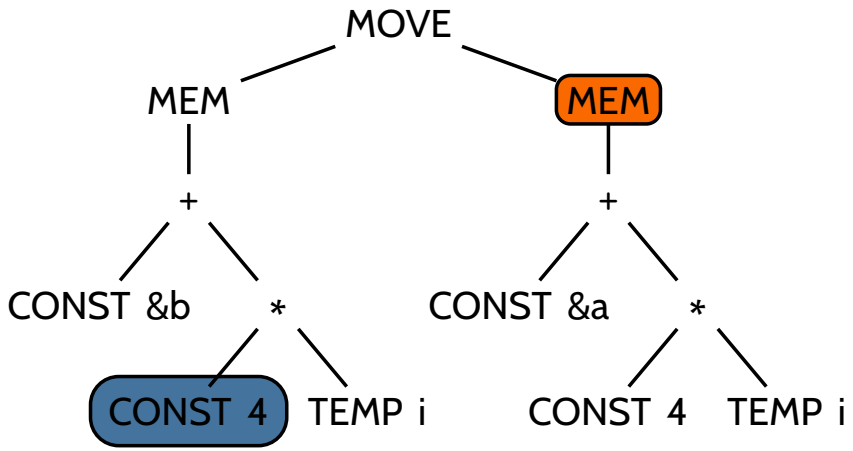
MOVE MEM + CONST &b * CONST 4 TEMP i

MEM + CONST &a * CONST 4 TEMP i $

Action: shift

Stack: MOVE MEM + CONST &b * T_0 TEMP i

Running tree parsing on our IR tree



Assembly code:

ADDI $t_0 \leftarrow r_0 + \#4$

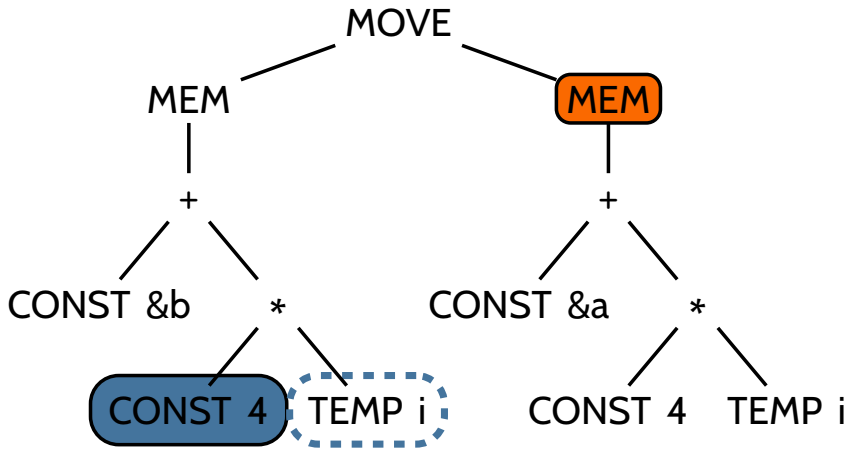
MOVE MEM + CONST &b * CONST 4 TEMP i

MEM + CONST &a * CONST 4 TEMP i $

Action: move to next symbol

Stack: MOVE MEM + CONST &b * T_0 TEMP i

Running tree parsing on our IR tree



Assembly code:

ADDI $t_0 \leftarrow r_0 + \#4$

MOVE MEM + CONST &b * CONST 4 TEMP i

MEM + CONST &a * CONST 4 TEMP i $

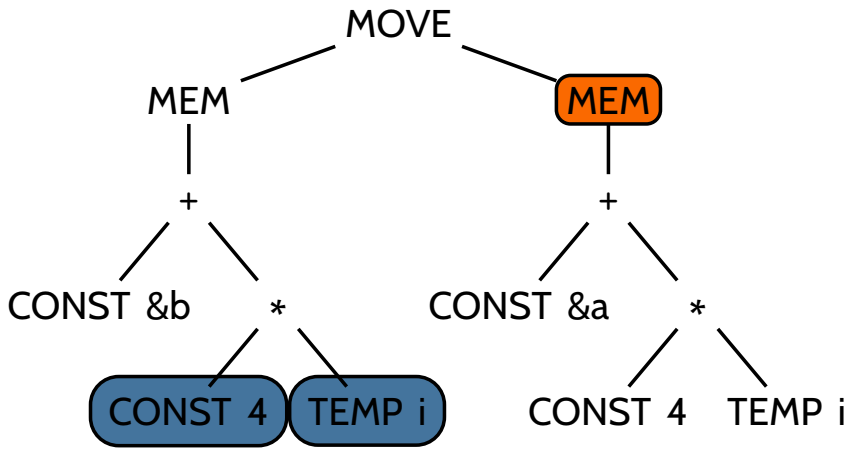
Action: reduce

Stack: MOVE MEM + CONST &b * T_0 TEMP i

$T_t \rightarrow$ TEMP t

due to $T_{new} \rightarrow * \; T_y \; T_z$

Running tree parsing on our IR tree



MOVE MEM + CONST &b * CONST 4 TEMP i

MEM + CONST &a * CONST 4 TEMP i $

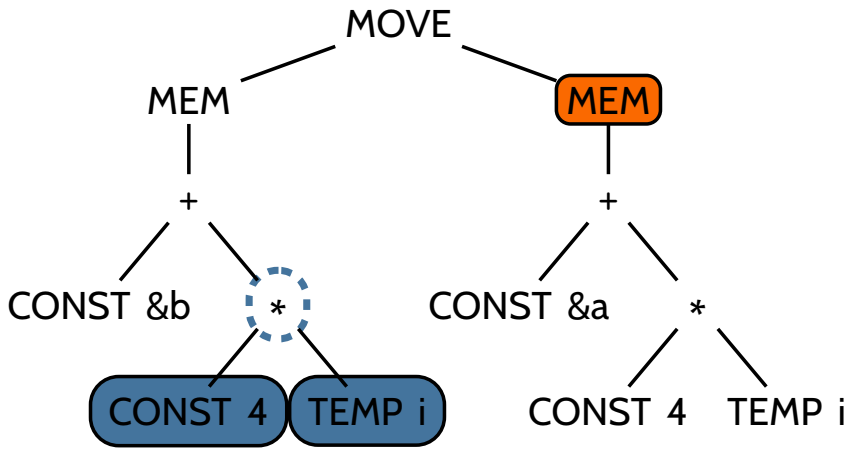
Action:

Stack: MOVE MEM + CONST &b * T_0 T_i

Assembly code:

ADDI $t_0 \leftarrow r_0 + \#4$

Running tree parsing on our IR tree



Assembly code:

ADDI $t_0 \leftarrow r_0 + \#4$

MOVE MEM + CONST &b * CONST 4 TEMP i

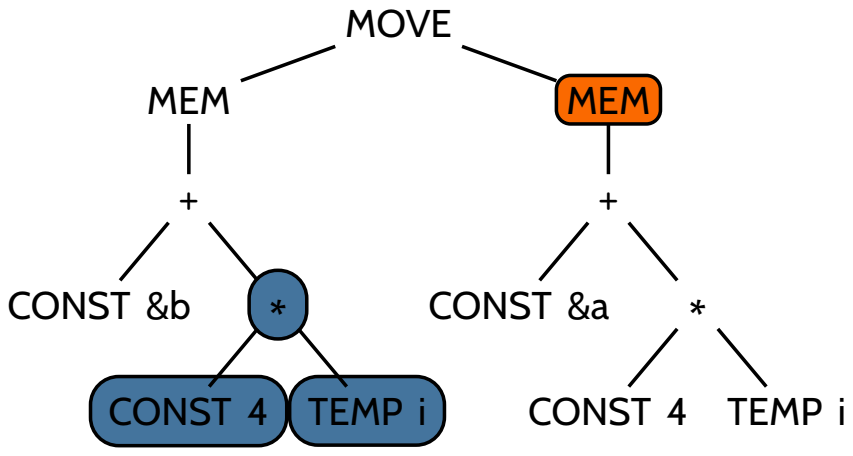
MEM + CONST &a * CONST 4 TEMP i $

Action: reduce

Stack: MOVE MEM + CONST &b * T_0 T_i

$T_{new} \rightarrow * \; T_y \; T_z$

Running tree parsing on our IR tree



Assembly code:

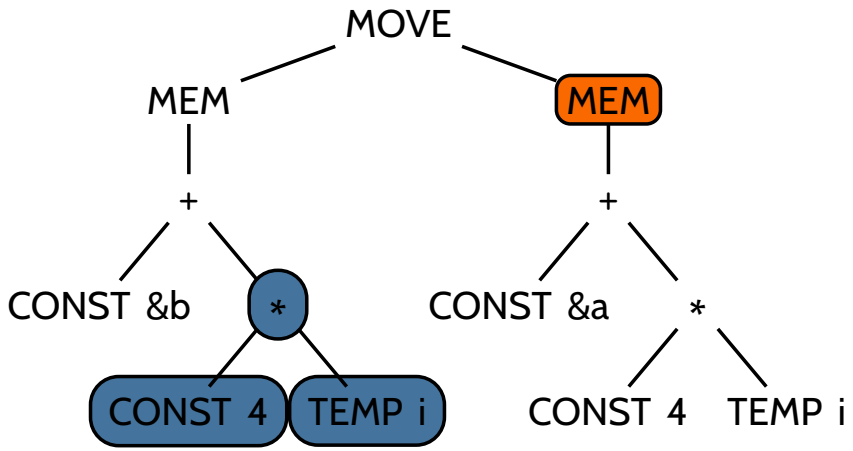
ADDI $t_0 \leftarrow r_0 + \#4$
MUL $t_1 \leftarrow t_0 * t_i$

MOVE MEM + CONST &b * CONST 4 TEMP i
MEM + CONST &a * CONST 4 TEMP i $

Action:

Stack: MOVE MEM + CONST &b T_1

Running tree parsing on our IR tree



Assembly code:

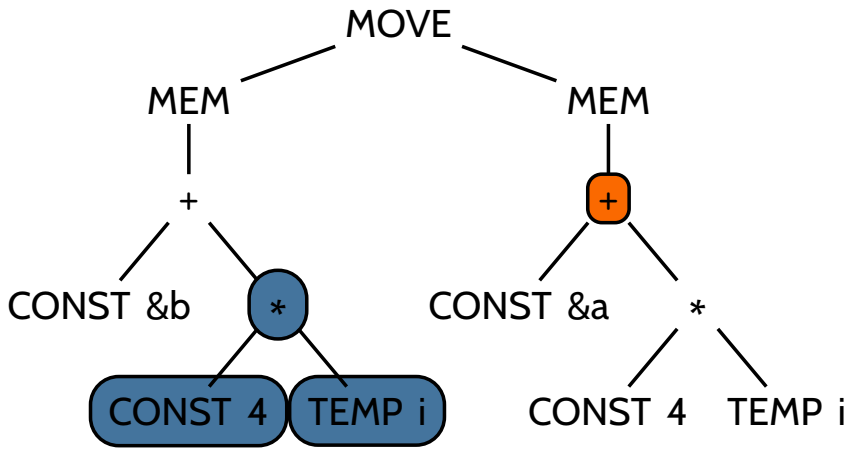
ADDI $t_0 \leftarrow r_0 + \#4$
MUL $t_1 \leftarrow t_0 * t_i$

MOVE MEM + CONST &b * CONST 4 TEMP i
MEM + CONST &a * CONST 4 TEMP i $

Action: shift

Stack: MOVE MEM + CONST &b T_1 MEM

Running tree parsing on our IR tree



MOVE MEM + CONST &b * CONST 4 TEMP i
MEM + CONST &a * CONST 4 TEMP i $

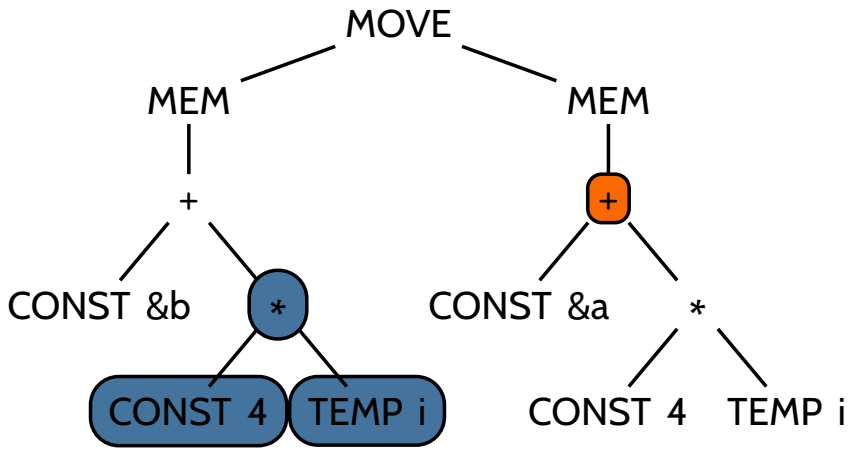
Action: move to next symbol

Stack: MOVE MEM + CONST &b T_1 MEM

Assembly code:

ADDI $t_0 \leftarrow r_0 + \#4$
MUL $t_1 \leftarrow t_0 * t_i$

Running tree parsing on our IR tree



Assembly code:

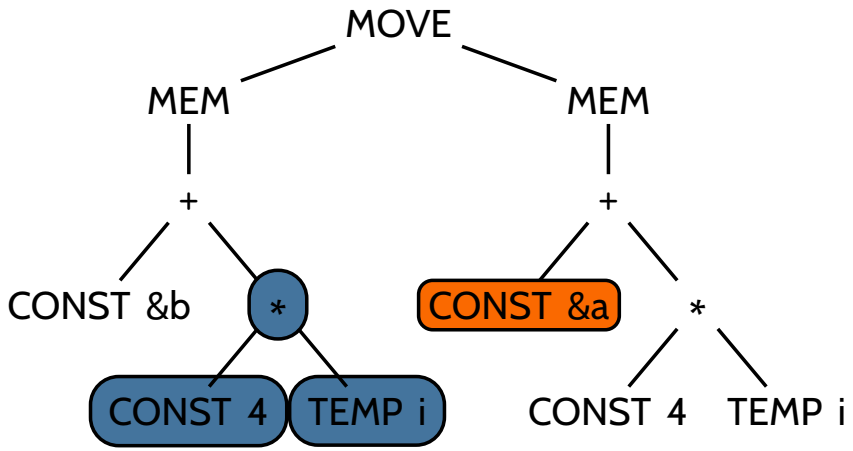
ADDI $t_0 \leftarrow r_0 + \#4$
MUL $t_1 \leftarrow t_0 * t_i$

MOVE MEM + CONST &b * CONST 4 TEMP i
MEM + CONST &a * CONST 4 TEMP i $

Action: shift

Stack: MOVE MEM + CONST &b T_1 MEM +

Running tree parsing on our IR tree



MOVE MEM + CONST &b * CONST 4 TEMP i

MEM + CONST &a * CONST 4 TEMP i $

Action: move to next symbol

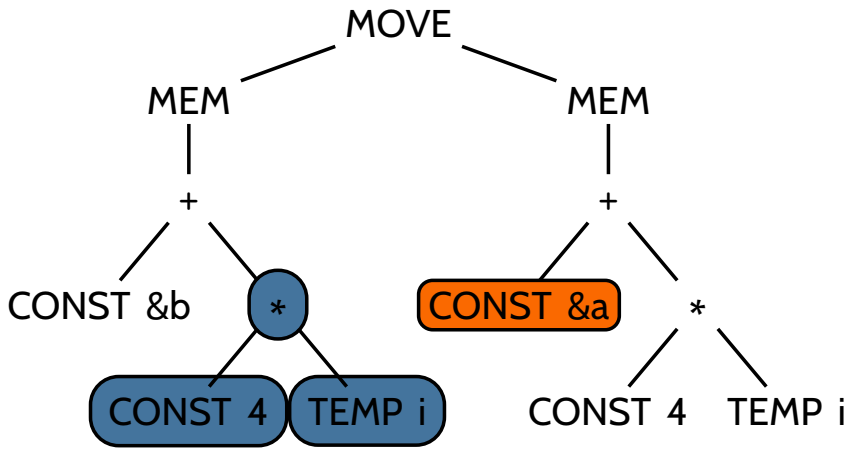
Stack: MOVE MEM + CONST &b T_1 MEM +

Assembly code:

ADDI $t_0 \leftarrow r_0 + \#4$

MUL $t_1 \leftarrow t_0 * t_i$

Running tree parsing on our IR tree



Assembly code:

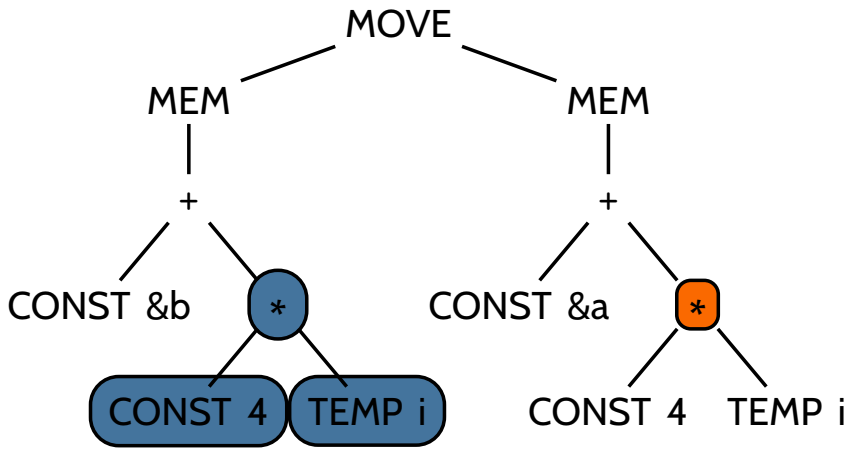
ADDI $t_0 \leftarrow r_0 + \#4$
MUL $t_1 \leftarrow t_0 * t_i$

MOVE MEM + CONST &b * CONST 4 TEMP i
MEM + CONST &a * CONST 4 TEMP i $

Action: shift

Stack: MOVE MEM + CONST &b T_1 MEM + CONST &a

Running tree parsing on our IR tree



Assembly code:

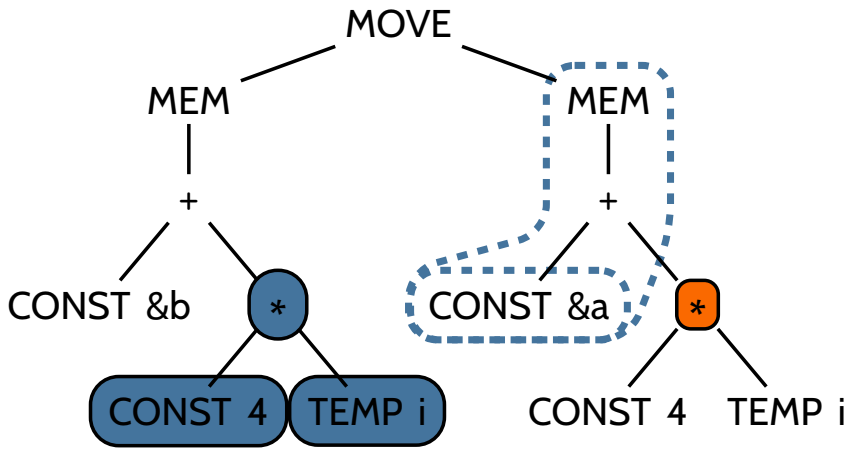
ADDI $t_0 \leftarrow r_0 + \#4$
MUL $t_1 \leftarrow t_0 * t_i$

MOVE MEM + CONST &b * CONST 4 TEMP i
MEM + CONST &a * CONST 4 TEMP i $

Action: move to next symbol

Stack: MOVE MEM + CONST &b T_1 MEM + CONST &a

Running tree parsing on our IR tree



Assembly code:

ADDI $t_0 \leftarrow r_0 + \#4$
MUL $t_1 \leftarrow t_0 * t_i$

MOVE MEM + CONST &b * CONST 4 TEMP i
MEM + CONST &a * CONST 4 TEMP i $

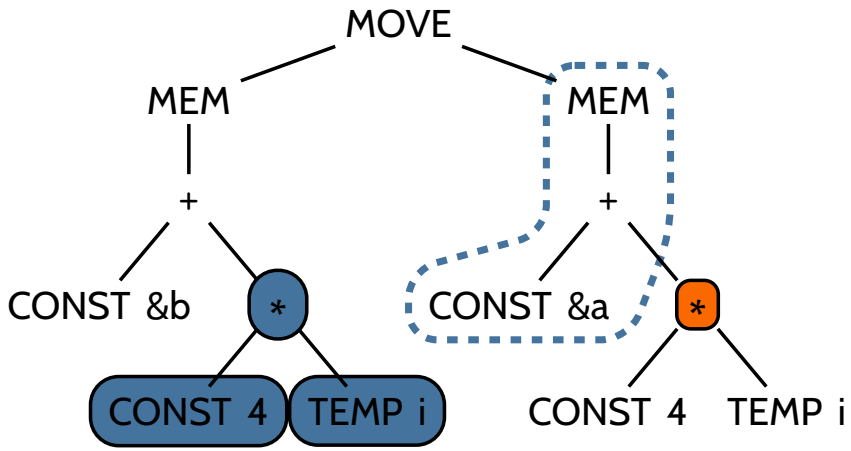
Action: shift-reduce conflict!

Stack: MOVE MEM + CONST &b T_1 MEM + CONST &a *

$T_{new} \rightarrow$ CONST c *reducible now*

$T_x \rightarrow$ MEM + CONST c T_y *may be reducible later*

Running tree parsing on our IR tree



Assembly code:

ADDI $t_0 \leftarrow r_0 + \#4$
MUL $t_1 \leftarrow t_0 * t_i$

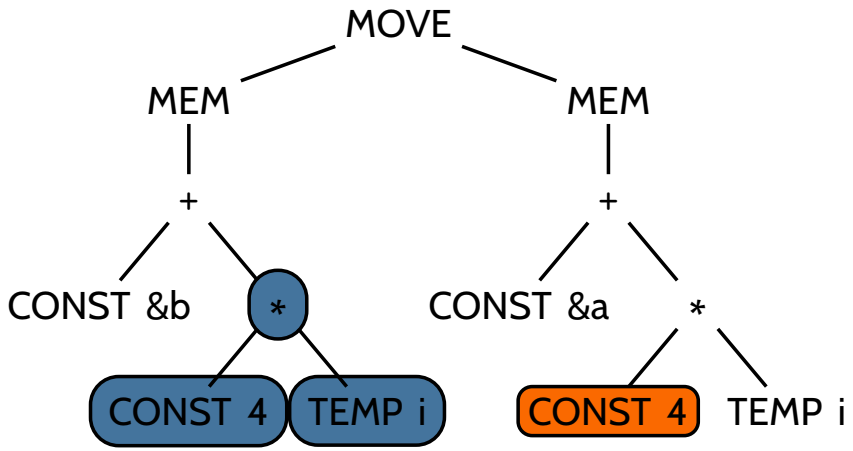
MOVE MEM + CONST &b * CONST 4 TEMP i
MEM + CONST &a * CONST 4 TEMP i $

Action: always shift

Stack: MOVE MEM + CONST &b T_1 MEM + CONST &a *

$T_x \rightarrow$ MEM + CONST c T_y *hope for later reduction*

Running tree parsing on our IR tree



Assembly code:

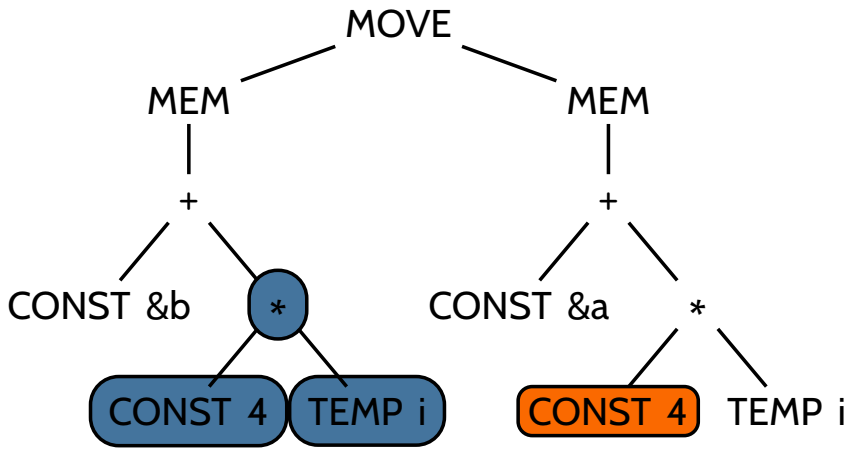
ADDI $t_0 \leftarrow r_0 + \#4$
MUL $t_1 \leftarrow t_0 * t_i$

MOVE MEM + CONST &b * CONST 4 TEMP i
MEM + CONST &a * CONST 4 TEMP i $

Action: move to next symbol

Stack: MOVE MEM + CONST &b T_1 MEM + CONST &a *

Running tree parsing on our IR tree



Assembly code:

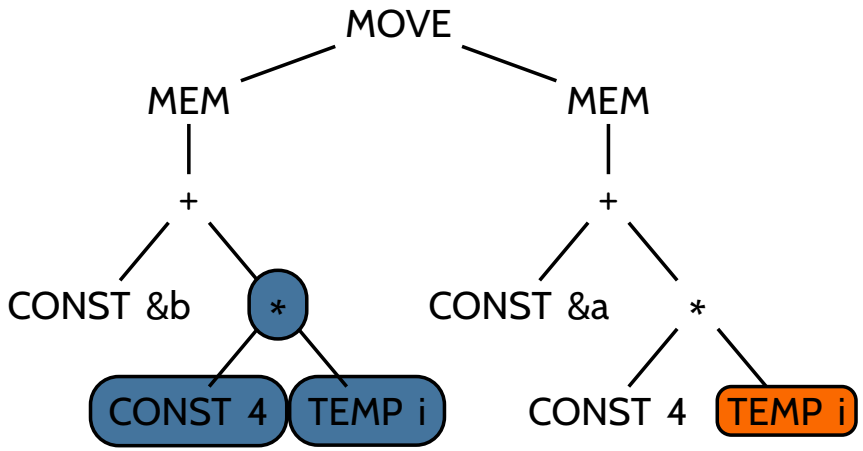
ADDI $t_0 \leftarrow r_0 + \#4$
MUL $t_1 \leftarrow t_0 * t_i$

MOVE MEM + CONST &b * CONST 4 TEMP i
MEM + CONST &a * CONST 4 TEMP i $

Action: shift

Stack: MOVE MEM + CONST &b T_1 MEM + CONST &a * CONST 4

Running tree parsing on our IR tree



Assembly code:

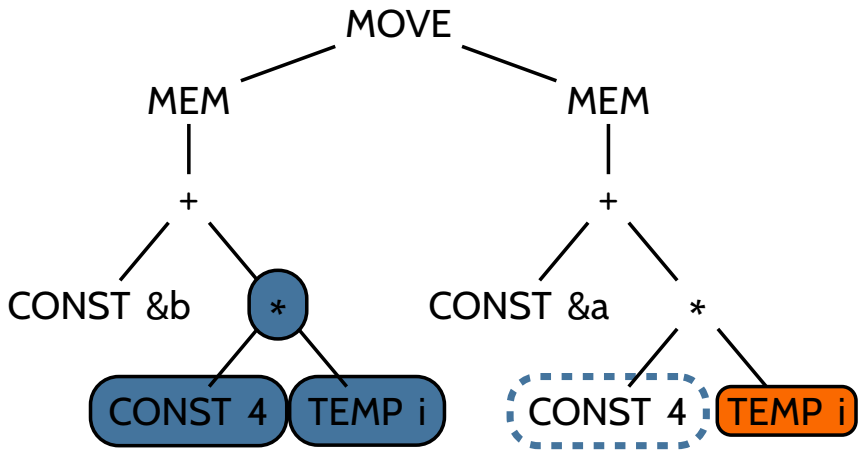
ADDI $t_0 \leftarrow r_0 + \#4$
MUL $t_1 \leftarrow t_0 * t_i$

MOVE MEM + CONST &b * CONST 4 TEMP i
MEM + CONST &a * CONST 4 TEMP i $

Action: move to next symbol

Stack: MOVE MEM + CONST &b T_1 MEM + CONST &a * CONST 4

Running tree parsing on our IR tree



Assembly code:

ADDI $t_0 \leftarrow r_0 + \#4$
MUL $t_1 \leftarrow t_0 * t_i$

MOVE MEM + CONST &b * CONST 4 TEMP i
MEM + CONST &a * CONST 4 TEMP i $

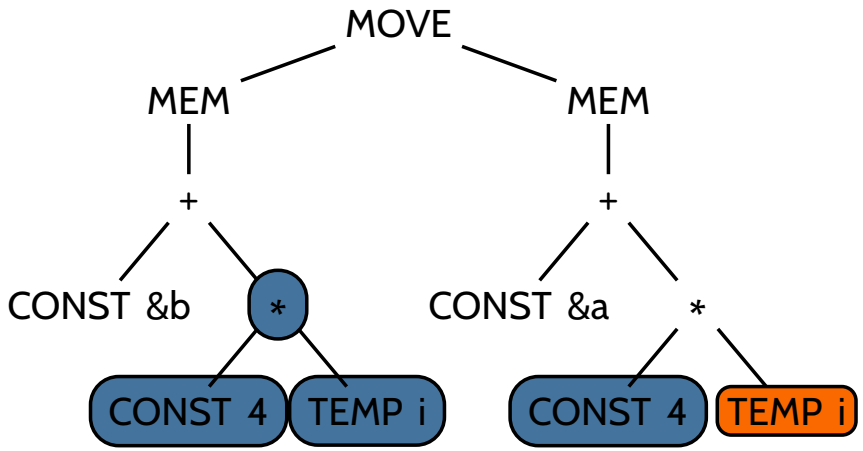
Action: reduce

Stack: MOVE MEM + CONST &b T_1 MEM + CONST &a * CONST 4

$T_{new} \rightarrow$ CONST c

due to $T_{new} \rightarrow * \; T_y \; T_z$

Running tree parsing on our IR tree



Assembly code:

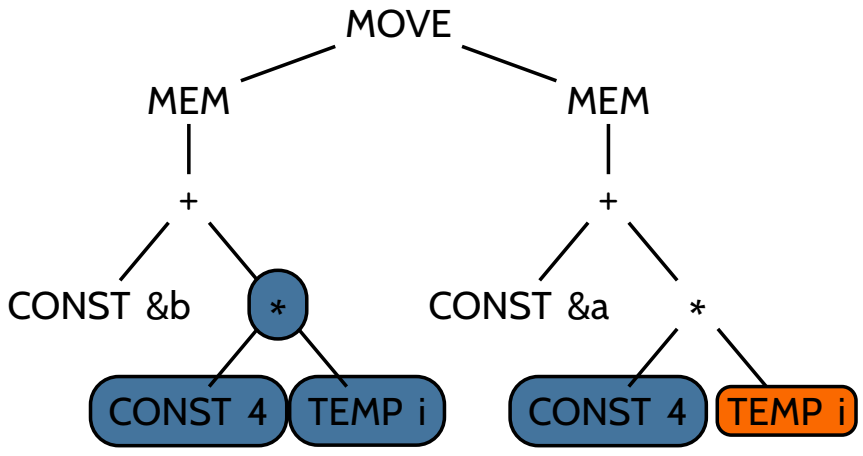
ADDI $t_0 \leftarrow r_0 + \#4$
MUL $t_1 \leftarrow t_0 * t_i$
ADDI $t_2 \leftarrow r_0 + \#4$

MOVE MEM + CONST &b * CONST 4 TEMP i
MEM + CONST &a * CONST 4 TEMP i $

Action:

Stack: MOVE MEM + CONST &b T_1 MEM + CONST &a * T_2

Running tree parsing on our IR tree



Assembly code:

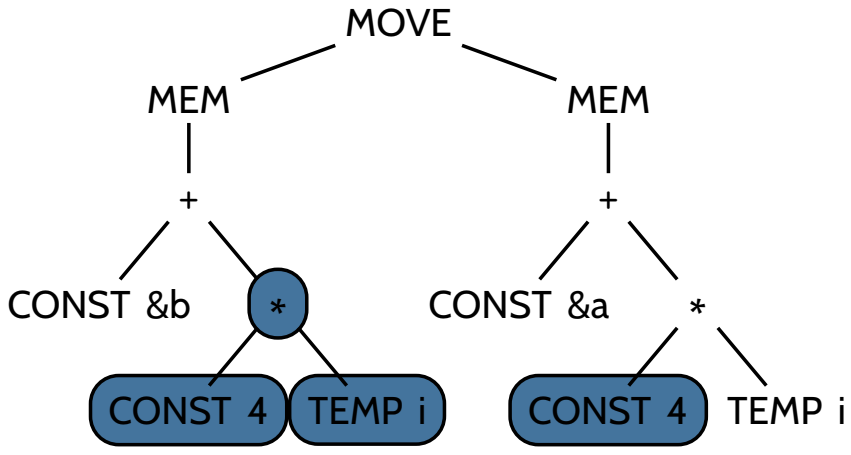
```
ADDI    t0 ← r0 + #4
MUL     t1 ← t0 * ti
ADDI    t2 ← r0 + #4
```

MOVE MEM + CONST &b * CONST 4 TEMP i
MEM + CONST &a * CONST 4 TEMP i $

Action: shift

Stack: MOVE MEM + CONST &b T_1 MEM + CONST &a * T_2 TEMP i

Running tree parsing on our IR tree



Assembly code:

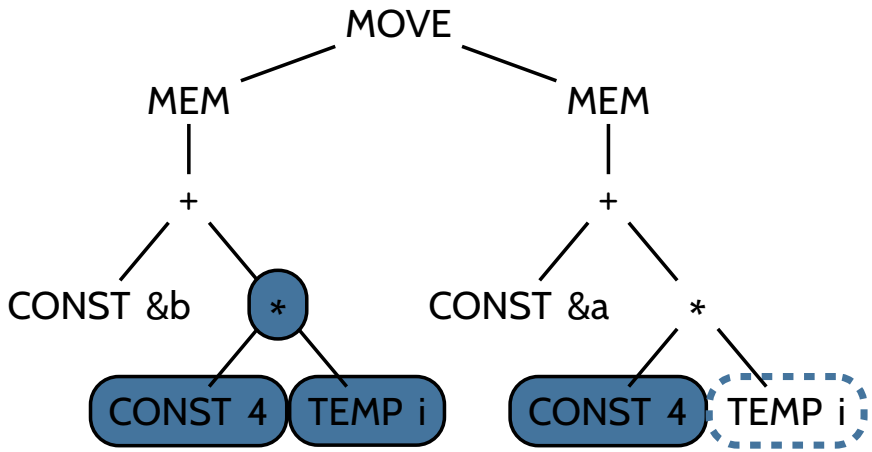
```
ADDI    t0 ← r0 + #4
MUL     t1 ← t0 * ti
ADDI    t2 ← r0 + #4
```

MOVE MEM + CONST &b * CONST 4 TEMP i
MEM + CONST &a * CONST 4 TEMP i $

Action: move to next symbol

Stack: MOVE MEM + CONST &b T_1 MEM + CONST &a * T_2 TEMP i

Running tree parsing on our IR tree



Assembly code:

```
ADDI    t0 ← r0 + #4
MUL     t1 ← t0 * ti
ADDI    t2 ← r0 + #4
```

MOVE MEM + CONST &b * CONST 4 TEMP i
MEM + CONST &a * CONST 4 TEMP i $

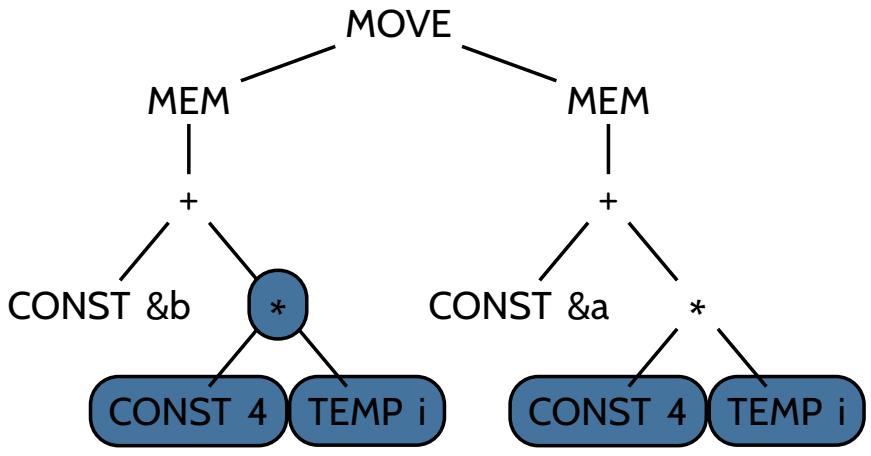
Action: reduce

Stack: MOVE MEM + CONST &b T_1 MEM + CONST &a * T_2 TEMP i

$T_t \rightarrow$ TEMP t

due to $T_{new} \rightarrow * \; T_y \; T_z$

Running tree parsing on our IR tree



Assembly code:

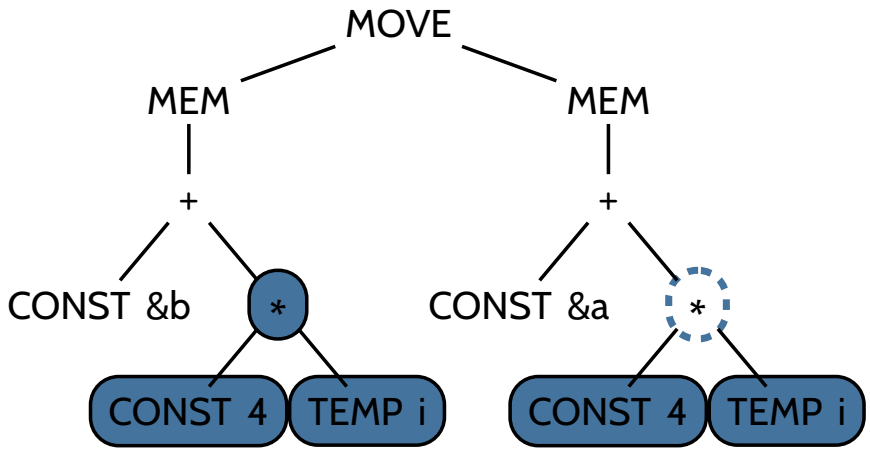
ADDI $t_0 \leftarrow r_0 + \#4$
MUL $t_1 \leftarrow t_0 * t_i$
ADDI $t_2 \leftarrow r_0 + \#4$

MOVE MEM + CONST &b * CONST 4 TEMP i
MEM + CONST &a * CONST 4 TEMP i $

Action:

Stack: MOVE MEM + CONST &b T_1 MEM + CONST &a * T_2 T_i

Running tree parsing on our IR tree



Assembly code:

ADDI $t_0 \leftarrow r_0 + \#4$
MUL $t_1 \leftarrow t_0 * t_i$
ADDI $t_2 \leftarrow r_0 + \#4$

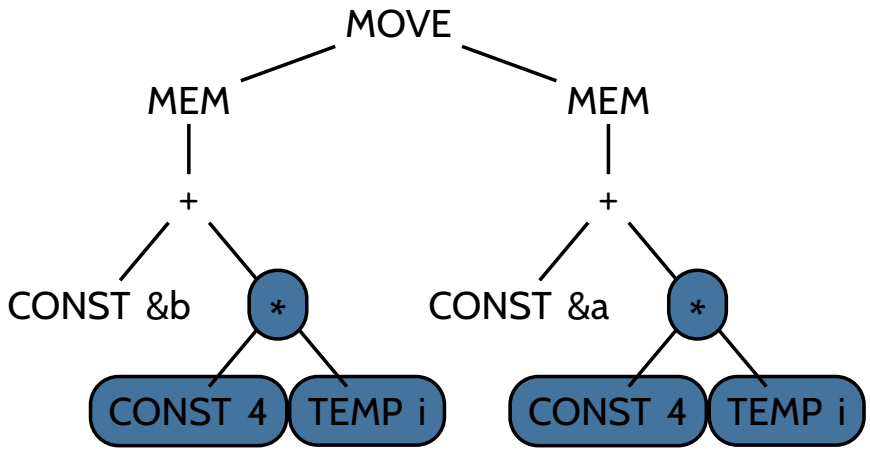
MOVE MEM + CONST &b * CONST 4 TEMP i
MEM + CONST &a * CONST 4 TEMP i $

Action: reduce

Stack: MOVE MEM + CONST &b T_1 MEM + CONST &a * T_2 T_i

$T_{new} \rightarrow * T_y T_z$

Running tree parsing on our IR tree



Assembly code:

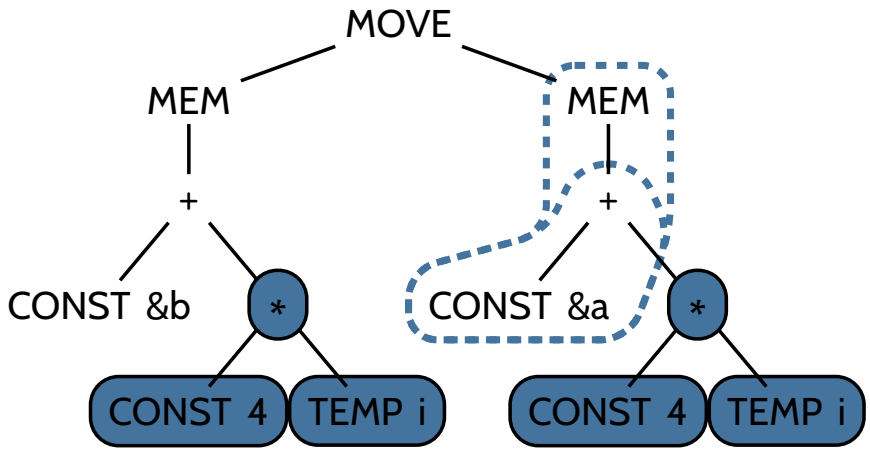
```
ADDI    t0 ← r0 + #4
MUL     t1 ← t0 * ti
ADDI    t2 ← r0 + #4
MUL     t3 ← t2 * ti
```

MOVE MEM + CONST &b * CONST 4 TEMP i
MEM + CONST &a * CONST 4 TEMP i $

Action:

Stack: MOVE MEM + CONST &b T_1 MEM + CONST &a T_3

Running tree parsing on our IR tree



Assembly code:

```
ADDI    t0 ← r0 + #4
MUL     t1 ← t0 * ti
ADDI    t2 ← r0 + #4
MUL     t3 ← t2 * ti
```

MOVE MEM + CONST &b * CONST 4 TEMP i
MEM + CONST &a * CONST 4 TEMP i $

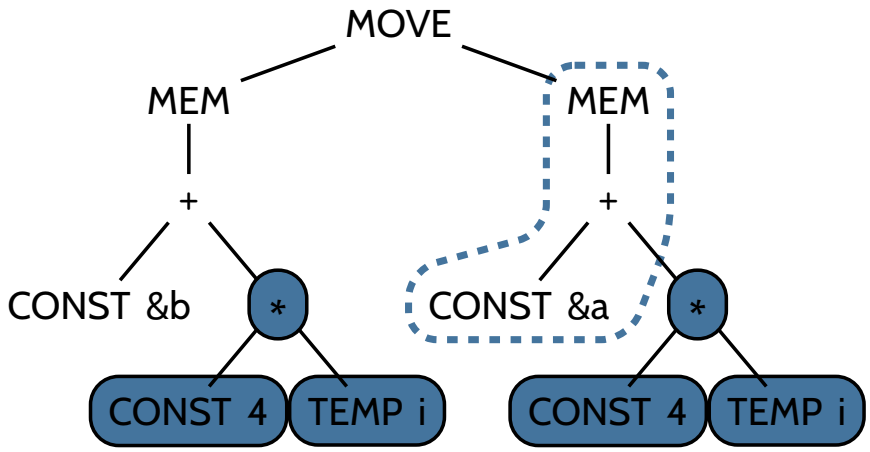
Action: reduce-reduce conflict!

Stack: MOVE MEM + CONST &b T_1 MEM + CONST &a T_3

$T_{new} \rightarrow$ + CONST c T_y

$T_{new} \rightarrow$ MEM + CONST c T_y

Running tree parsing on our IR tree



Assembly code:

```
ADDI   t0 ← r0 + #4
MUL    t1 ← t0 * ti
ADDI   t2 ← r0 + #4
MUL    t3 ← t2 * ti
```

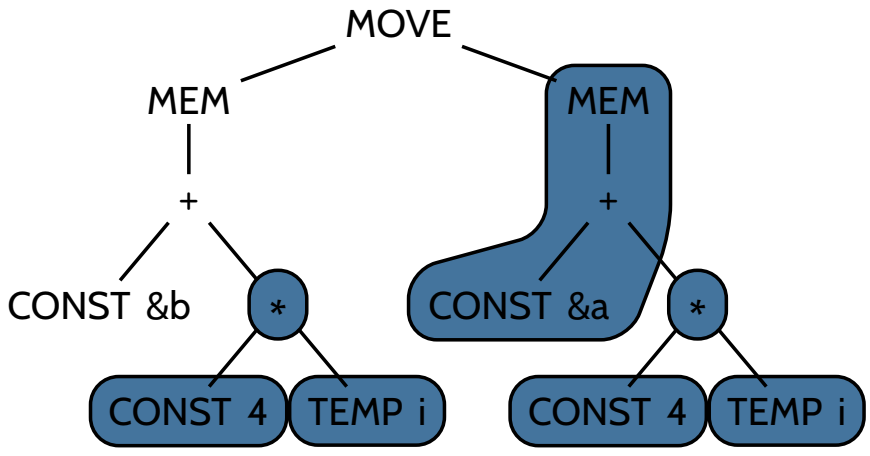
MOVE MEM + CONST &b * CONST 4 TEMP i
MEM + CONST &a * CONST 4 TEMP i $

Action: reduce longest production

Stack: MOVE MEM + CONST &b T_1 MEM + CONST &a T_3

T_{new} → MEM + CONST c T_y

Running tree parsing on our IR tree



Assembly code:

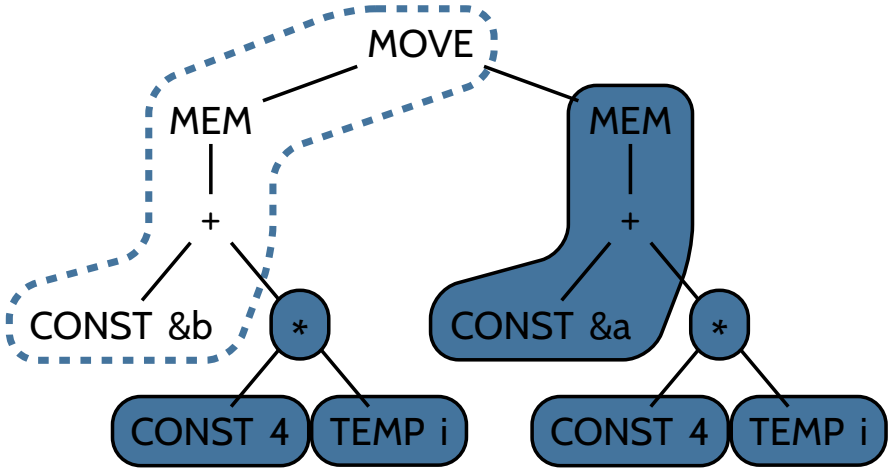
```
ADDI   t0 ← r0 + #4
MUL    t1 ← t0 * ti
ADDI   t2 ← r0 + #4
MUL    t3 ← t2 * ti
LOAD   t4 ← M[t3 + #&a]
```

MOVE MEM + CONST &b * CONST 4 TEMP i
MEM + CONST &a * CONST 4 TEMP i $

Action:

Stack: MOVE MEM + CONST &b T_1 T_4

Running tree parsing on our IR tree



MOVE MEM + CONST &b * CONST 4 TEMP i
MEM + CONST &a * CONST 4 TEMP i $

Action: reduce

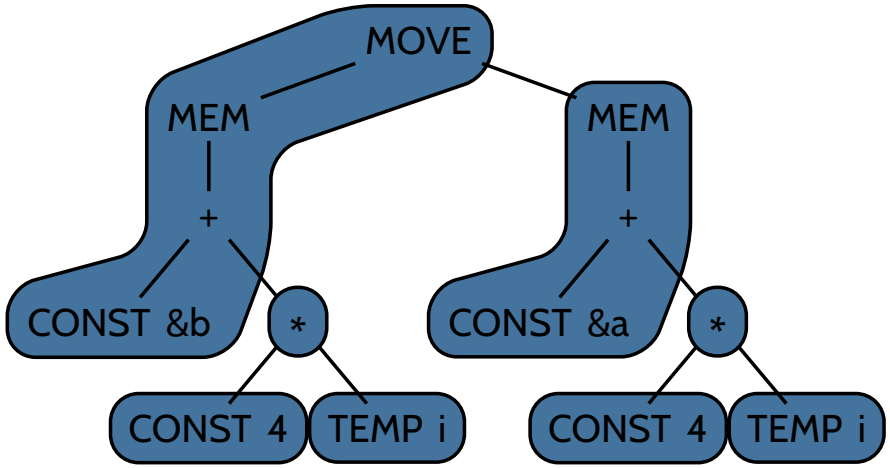
Stack: MOVE MEM + CONST &b T_1 T_4

$T \rightarrow$ MOVE MEM + CONST c T_x T_y

Assembly code:

```
ADDI   t0 ← r0 + #4
MUL    t1 ← t0 * ti
ADDI   t2 ← r0 + #4
MUL    t3 ← t2 * ti
LOAD   t4 ← M[t3 + #&a]
```

Running tree parsing on our IR tree



MOVE MEM + CONST &b * CONST 4 TEMP i
MEM + CONST &a * CONST 4 TEMP i $

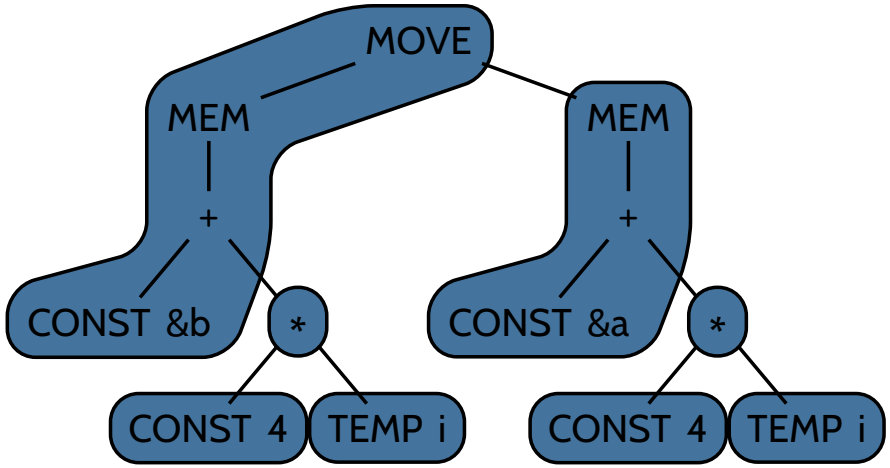
Action:

Stack: *T*

Assembly code:

```
ADDI    t0 ← r0 + #4
MUL     t1 ← t0 * ti
ADDI    t2 ← r0 + #4
MUL     t3 ← t2 * ti
LOAD    t4 ← M[t3 + #&a]
STORE   M[t1 + #&b] ← t4
```

Running tree parsing on our IR tree



MOVE MEM + CONST &b * CONST 4 TEMP i
MEM + CONST &a * CONST 4 TEMP i $

Action: accept

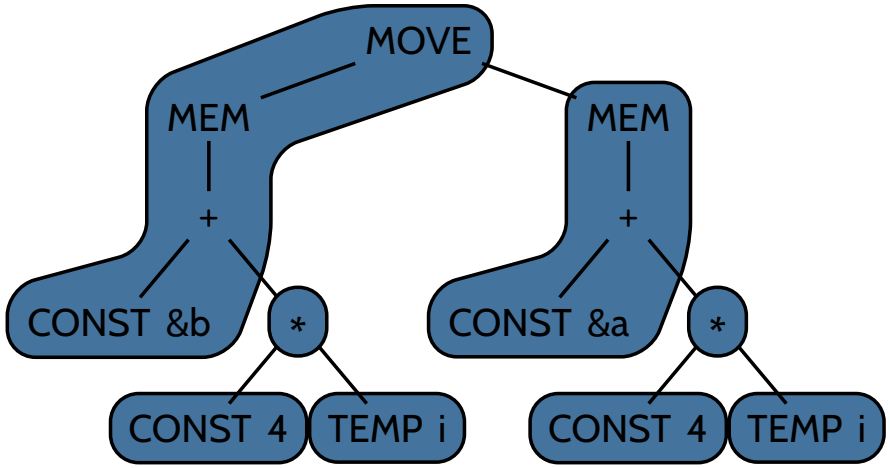
Stack: *T*

S → *T* $

Assembly code:

ADDI $t_0 \leftarrow r_0 + \#4$
MUL $t_1 \leftarrow t_0 * t_i$
ADDI $t_2 \leftarrow r_0 + \#4$
MUL $t_3 \leftarrow t_2 * t_i$
LOAD $t_4 \leftarrow M[t_3 + \#\&a]$
STORE $M[t_1 + \#\&b] \leftarrow t_4$

Running tree parsing on our IR tree



MOVE MEM + CONST &b * CONST 4 TEMP i
MEM + CONST &a * CONST 4 TEMP i $

Action: done

Stack:

Assembly code:

ADDI $t_0 \leftarrow r_0 + \#4$
MUL $t_1 \leftarrow t_0 * t_i$
ADDI $t_2 \leftarrow r_0 + \#4$
MUL $t_3 \leftarrow t_2 * t_i$
LOAD $t_4 \leftarrow M[t_3 + \#\&a]$
STORE $M[t_1 + \#\&b] \leftarrow t_4$

Limitation: Tree parsing could fail

- **Syntactic blocking:**
  - Always shifting in shift-reduce conflicts is a *guess* that may prove wrong
- **Stack at conflict:**

Stack: . . . MEM + CONST 4 *

$T_{new} \rightarrow$ CONST c *reducible now*

$T \rightarrow$ MEM + CONST c T_x T_y *may be reducible later*

- **Stack some time after shifting:**

Stack: . . . MEM + CONST 4 T_5 X

$T \rightarrow$ MEM + CONST c T_x T_y *no longer reducible!*

- **Solution:**
  - Add auxiliary productions that fix the stack (in other words, "undo" erroneous guesses)

Quality of emitted assembly code

		Costs:
ADDI	$t_0 \leftarrow r_0 + \#4$	1
MUL	$t_1 \leftarrow t_0 * t_i$	2
ADDI	$t_2 \leftarrow r_0 + \#4$	1
MUL	$t_3 \leftarrow t_2 * t_i$	2
LOAD	$t_4 \leftarrow M[t_3 + \#\&a]$	10
STORE	$M[t_1 + \#\&b] \leftarrow t_4$	10

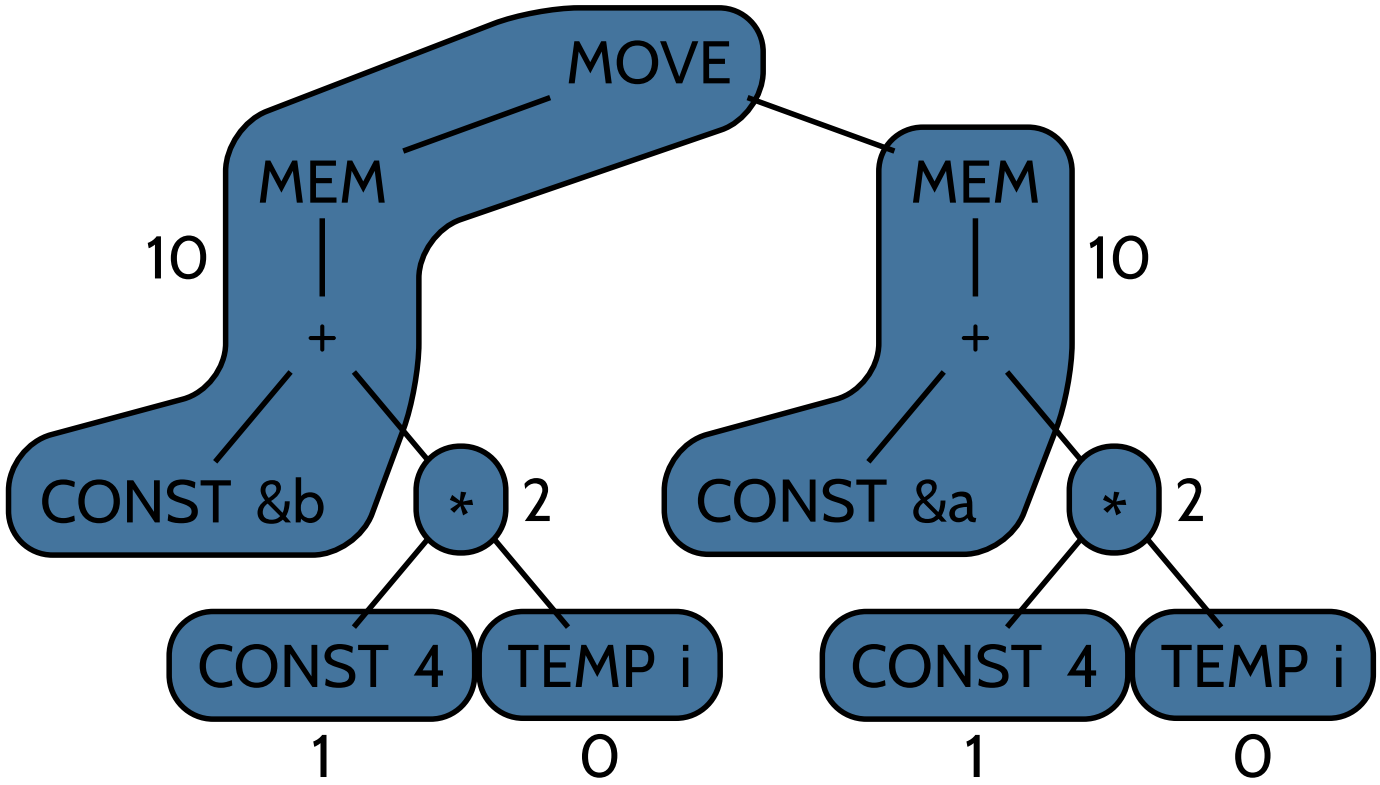
$$\sum \text{cost} = 26$$

Can we do better?

		Costs:
ADDI	$t_0 \leftarrow r_0 + \#4$	1
MUL	$t_1 \leftarrow t_0 * t_i$	2
ADDI	$t_2 \leftarrow r_0 + \#4$	1
MUL	$t_3 \leftarrow t_2 * t_i$	2
LOAD	$t_4 \leftarrow M[t_3 + \#\&a]$	10
STORE	$M[t_1 + \#\&b] \leftarrow t_4$	10

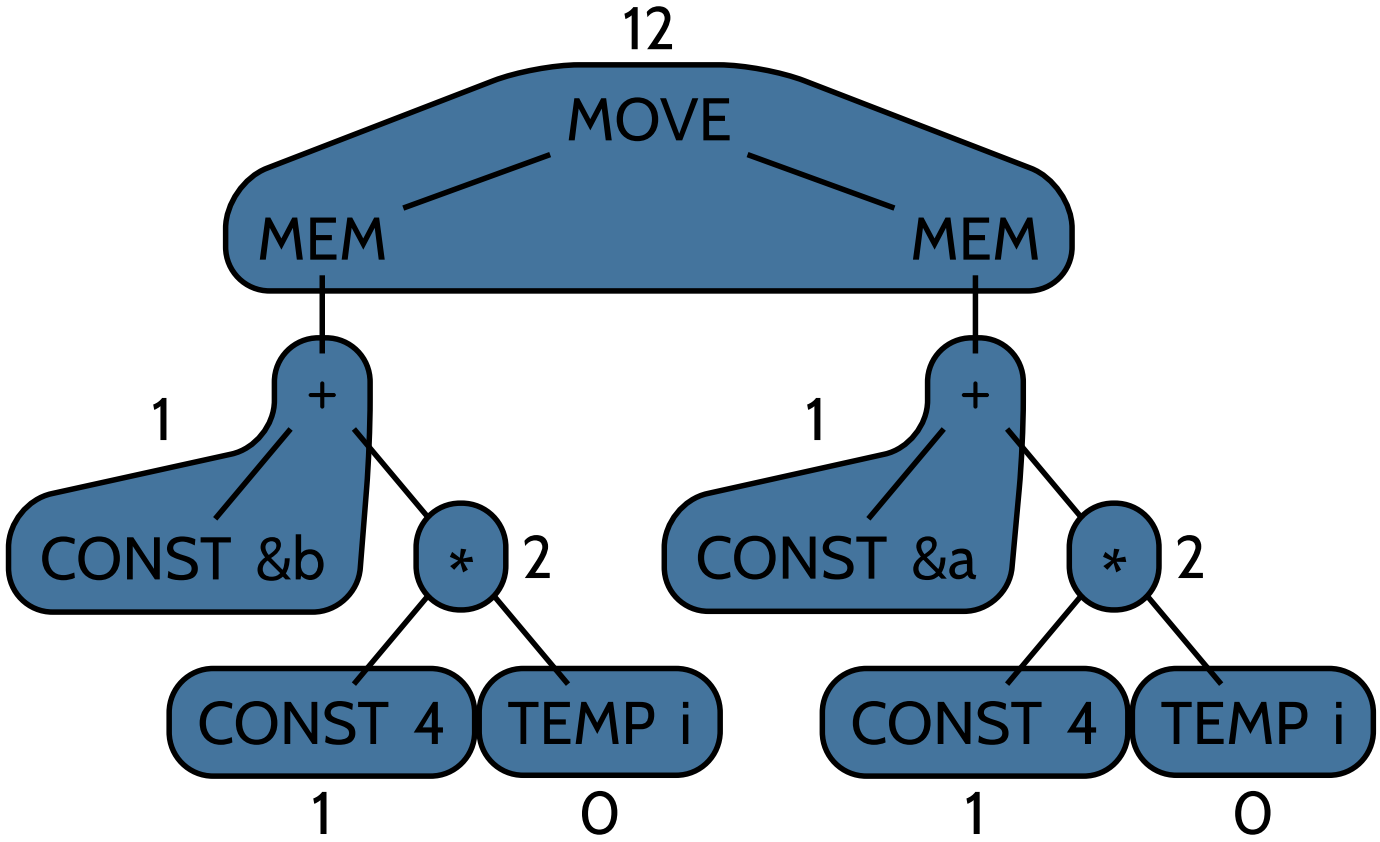
$$\sum \text{cost} = 26$$

Optimal tiling found with tree parsing



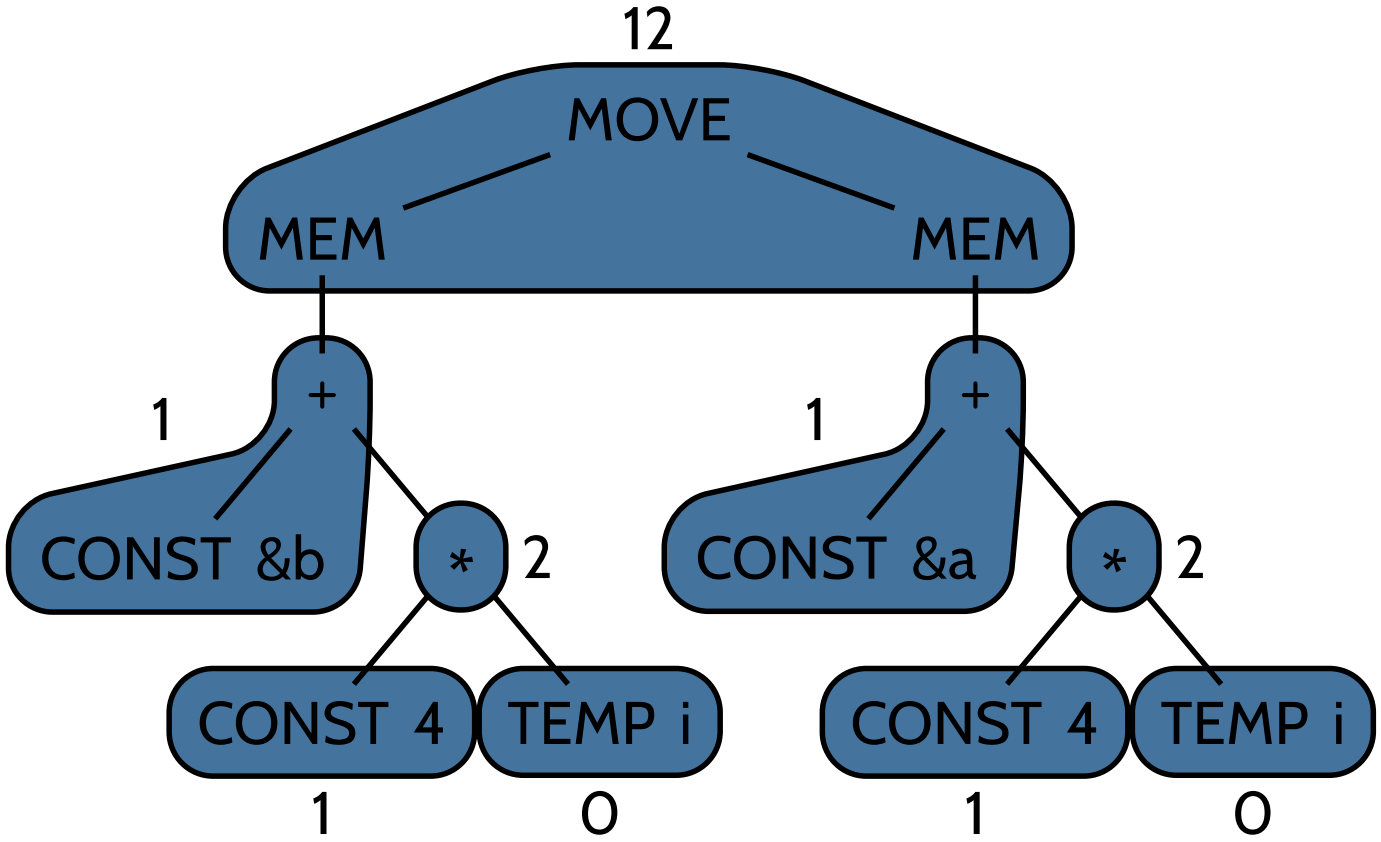
$$\sum \text{cost} = 26$$

Optimum tiling



$$\sum \text{cost} = 20$$

Need non-greedy approaches to find this tiling



$$\sum \text{cost} = 20$$

4th approach:

Dynamic Programming

Fundamental idea

- Derive tree grammar from tile set
- To find optimum tiling:
  1. Find all tiles that match IR tree
  2. Traverse IR tree bottom up:
     - Record least cost of reducing current IR operation to a particular nonterminal
  3. Traverse IR tree top down:
     - Select production that produces nonterminal at least cost
     - Repeat for all subtrees
- To emit assembly code:
  - Traverse IR tree bottom up
  - For each tile in tiling:
    - Emit corresponding instruction

Finding tiles that match

- Perform **bottom-up tree labeling**
  - See paper by Hoffmann & O'Donnell "Pattern Matching in Trees" (1982)
    `http://dx.doi.org/10.1145/322290.322295`
- Can be done in linear time $\mathcal{O}(n)$

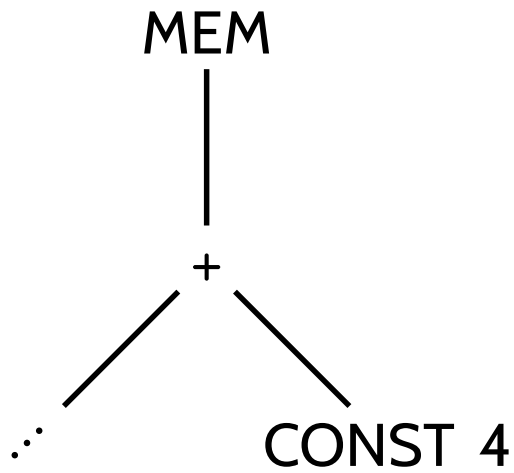
Cost of reduction

- Cost of reducing nonterminal *nt* using production *P*:

$$c_P + \sum_{1}^{n} c_i$$

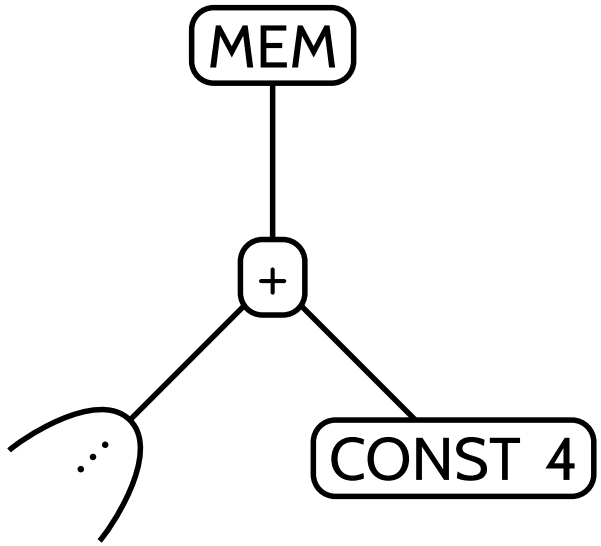
c_P = cost of *P* itself
c_i = cost of *i*th nonterminal appearing in *P*

Computing costs on example



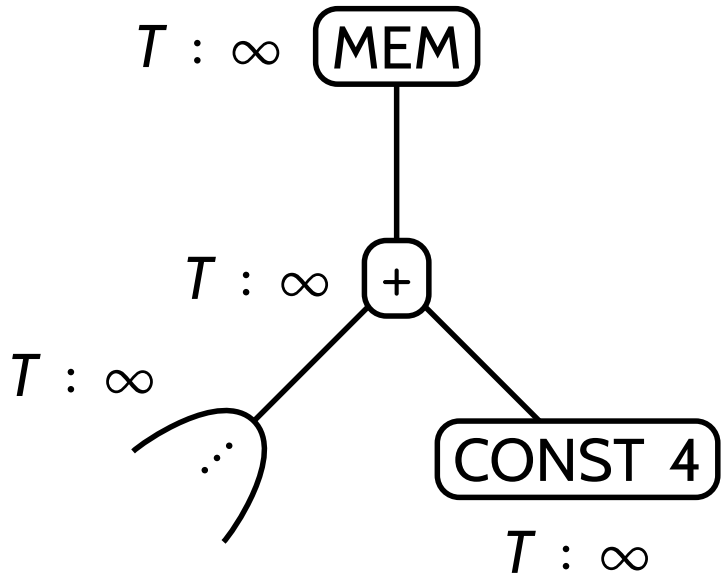
Action:

Computing costs on example



Action: added boxes for better readability

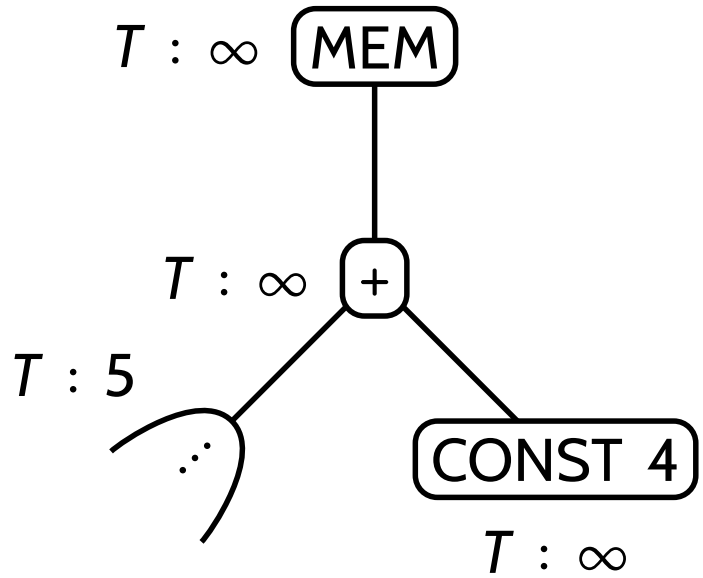
Computing costs on example



Action: initialize all costs to ∞

- $T : c$ denotes "reducible to nonterminal T at cost c"

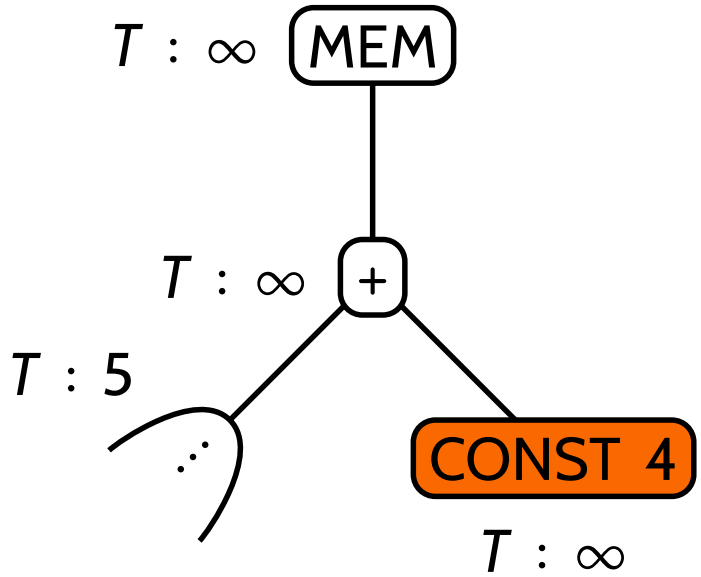
Computing costs on example



Action: assume $T : 5$ already found for subtree

- $T : c$ denotes "reducible to nonterminal T at cost c"

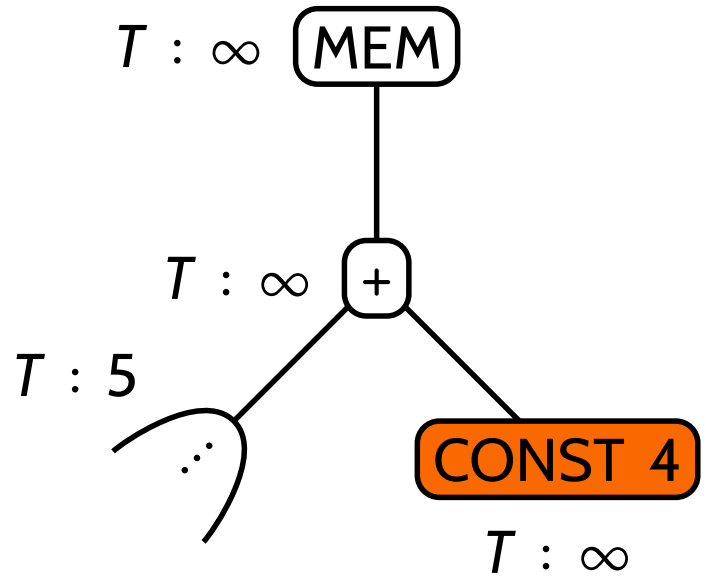
Computing costs on example



Action: start at CONST 4 node

- $T : c$ denotes "reducible to nonterminal T at cost c"

Computing costs on example

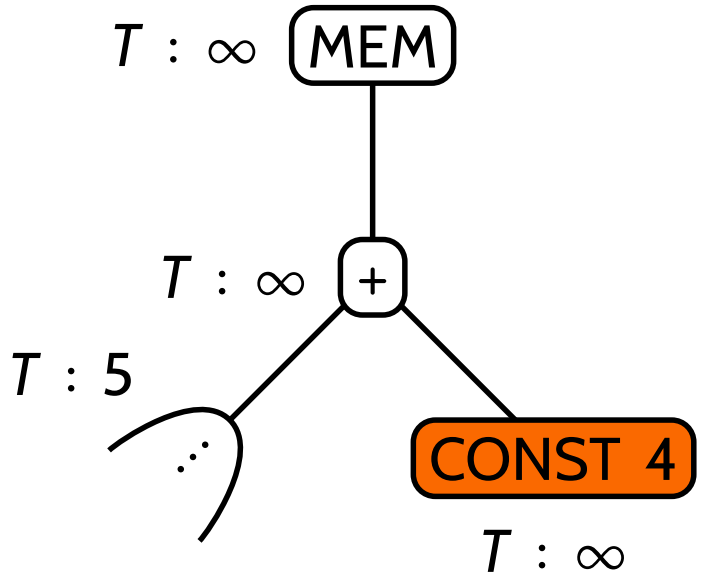


Productions:	**Costs:**
(T_{new} → CONST c)	1

Action: get productions of tiles that match

- $T : c$ denotes "reducible to nonterminal T at cost c"

Computing costs on example



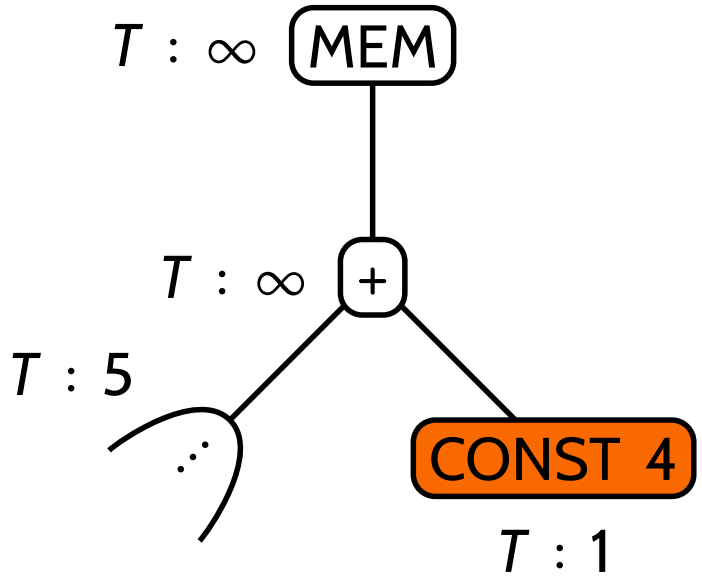
Productions: ($T_{\text{new}} \rightarrow$ CONST c)

Costs: $1 + 0$

Action: compute costs of reduction

- $T : c$ denotes "reducible to nonterminal T at cost c"

Computing costs on example



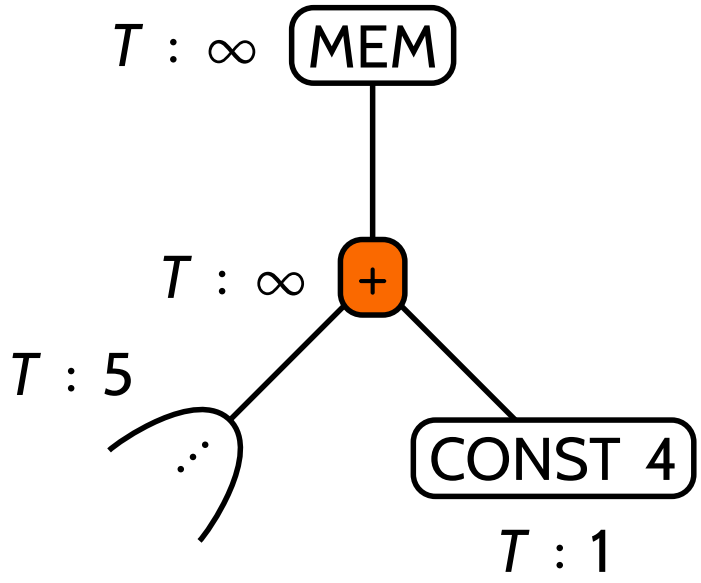
Productions: **Costs:**

$(T_{new} \rightarrow \text{CONST } c)$	1

Action: select production with least cost

- $T : c$ denotes "reducible to nonterminal T at cost c"

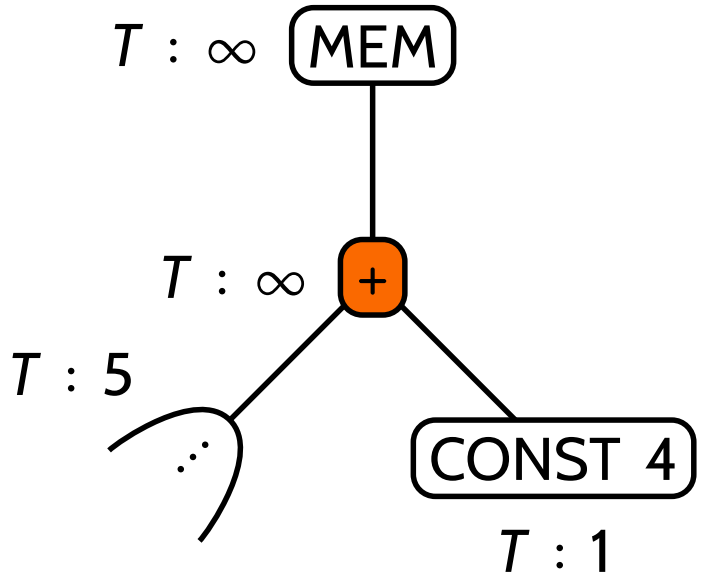
Computing costs on example



Action: continue to + node

- $T : c$ denotes "reducible to nonterminal T at cost c"

Computing costs on example

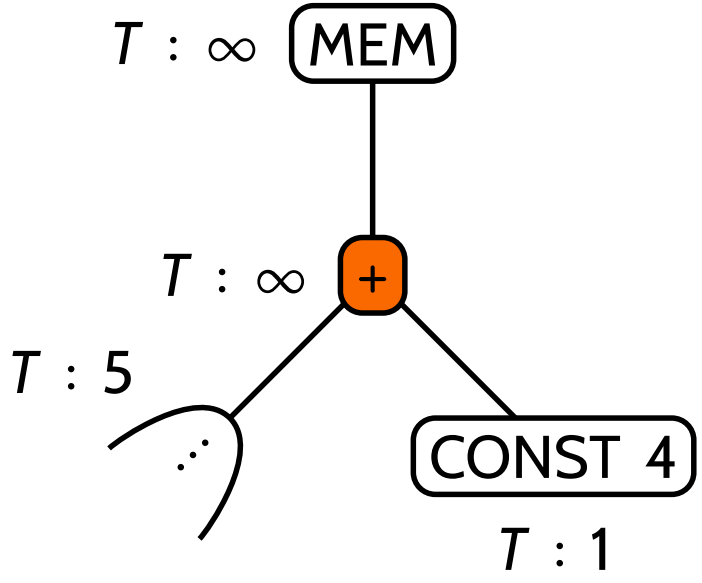


Productions:	**Costs:**
$(T_{\text{new}} \rightarrow + \; T_y \; T_z)$	1
$(T_{\text{new}} \rightarrow + \; T_y \; \text{CONST} \; c)$	1

Action: get productions of tiles that match

- $T : c$ denotes "reducible to nonterminal T at cost c"

Computing costs on example

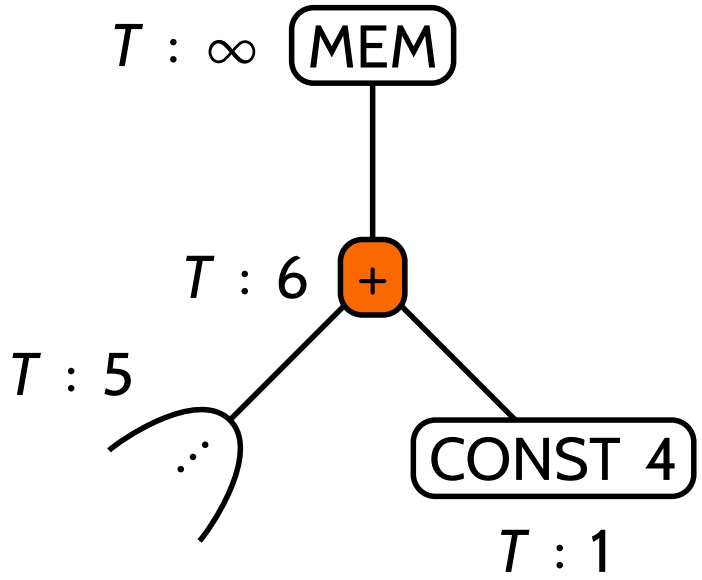


Productions:	**Costs:**
$(T_{\text{new}} \rightarrow + \; T_y \; T_z)$	$1 + 5 + 1$
$(T_{\text{new}} \rightarrow + \; T_y \; \text{CONST} \; c)$	$1 + 5$

Action: compute costs of reduction

- $T : c$ denotes "reducible to nonterminal T at cost c"

Computing costs on example

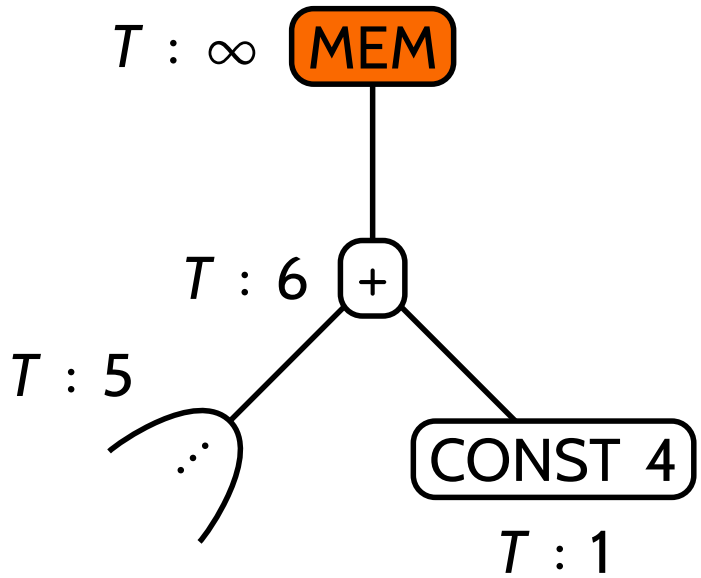


Productions:	**Costs:**
$(T_{\text{new}} \rightarrow + \; T_y \; T_z)$	7
$(T_{\text{new}} \rightarrow + \; T_y \; \text{CONST } c)$	6

Action: select production with least cost

- $T : c$ denotes "reducible to nonterminal T at cost c"

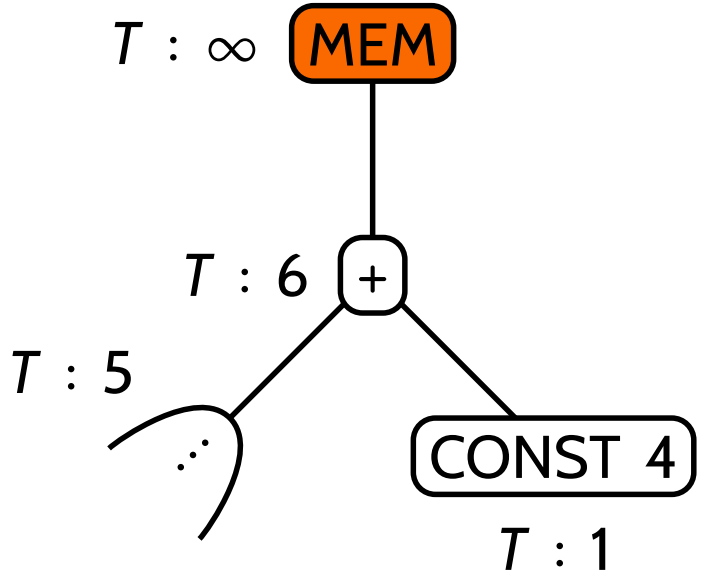
Computing costs on example



Action: continue to MEM node

- $T : c$ denotes "reducible to nonterminal T at cost c"

Computing costs on example

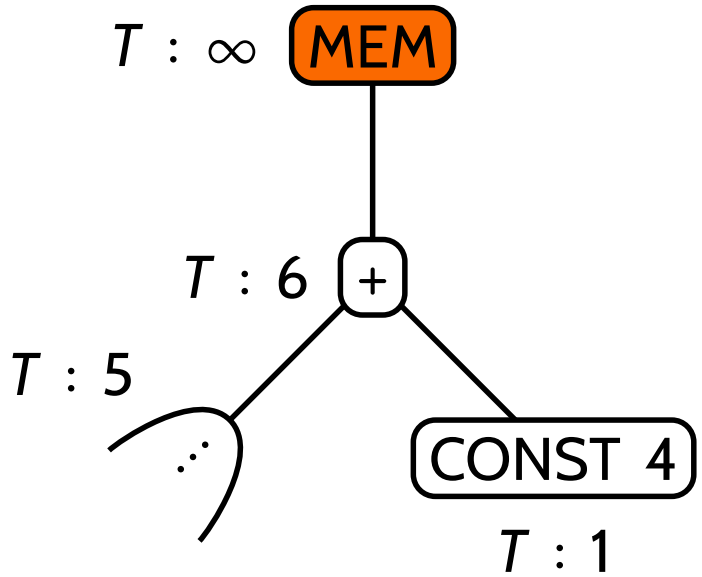


Productions:	**Costs:**
$(T_{\text{new}} \rightarrow \text{MEM } T_y)$	10
$(T_{\text{new}} \rightarrow \text{MEM + } T_y \text{ CONST } c)$	10

Action: get productions of tiles that match

- $T : c$ denotes "reducible to nonterminal T at cost c"

Computing costs on example

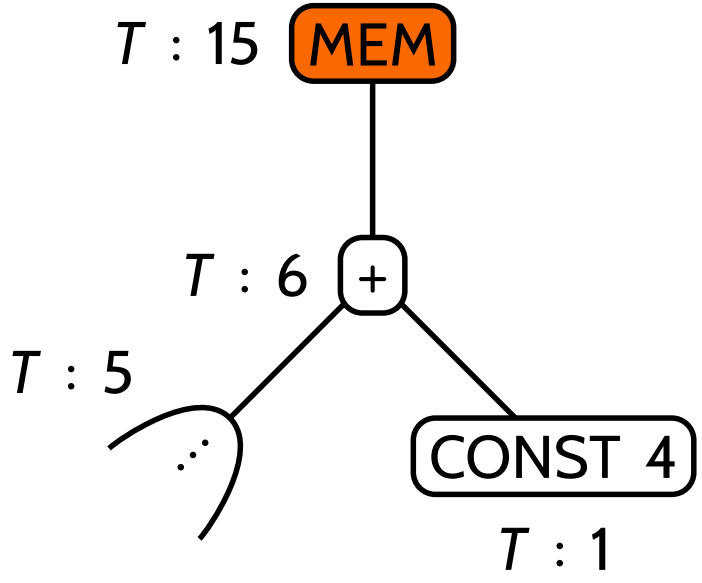


Productions:	**Costs:**
$(T_{\text{new}} \rightarrow \text{MEM } T_y)$	$10 + 6$
$(T_{\text{new}} \rightarrow \text{MEM} + T_y \text{ CONST } c)$	$10 + 5$

Action: compute costs of reduction

- $T : c$ denotes "reducible to nonterminal T at cost c"

Computing costs on example

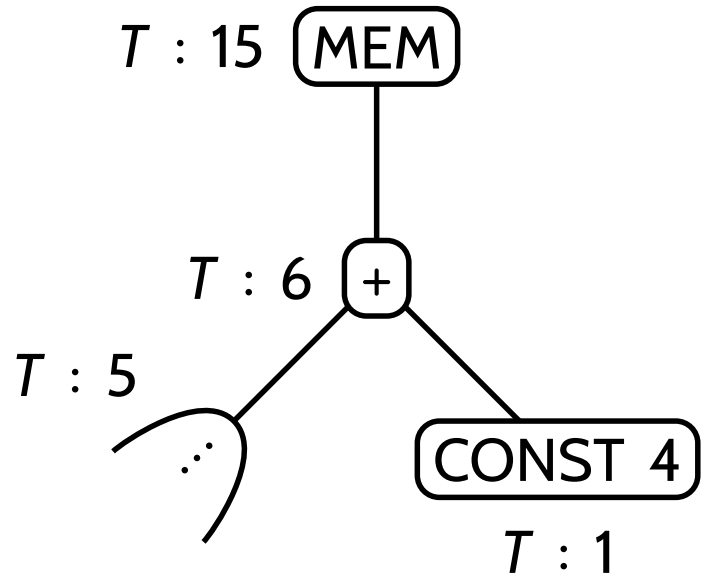


Productions:	**Costs:**
$(T_{\text{new}} \rightarrow \text{MEM } T_y)$	16
$(T_{\text{new}} \rightarrow \text{MEM + } T_y \text{ CONST } c)$	15

Action: select production with least cost

- $T : c$ denotes "reducible to nonterminal T at cost c"

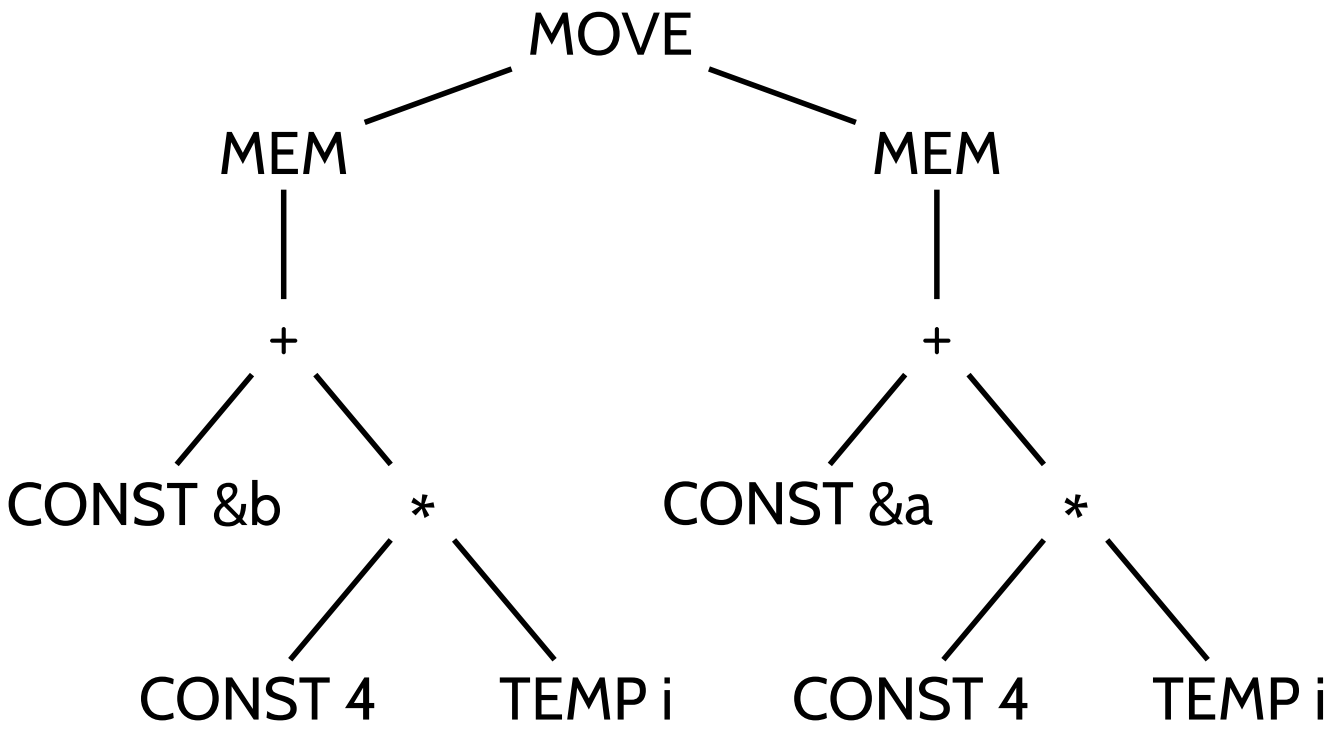
Computing costs on example



Action: done

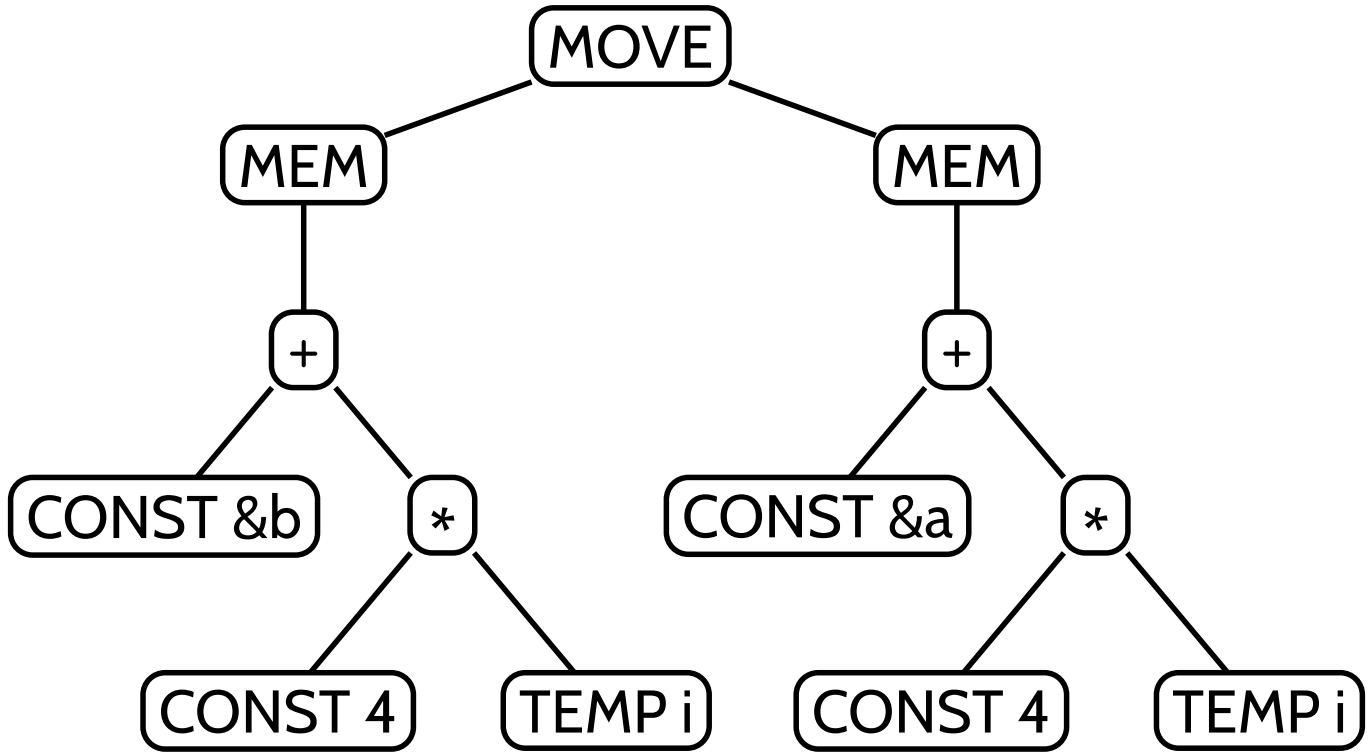
- $T : c$ denotes "reducible to nonterminal T at cost c"

Running dynamic programming on our IR tree



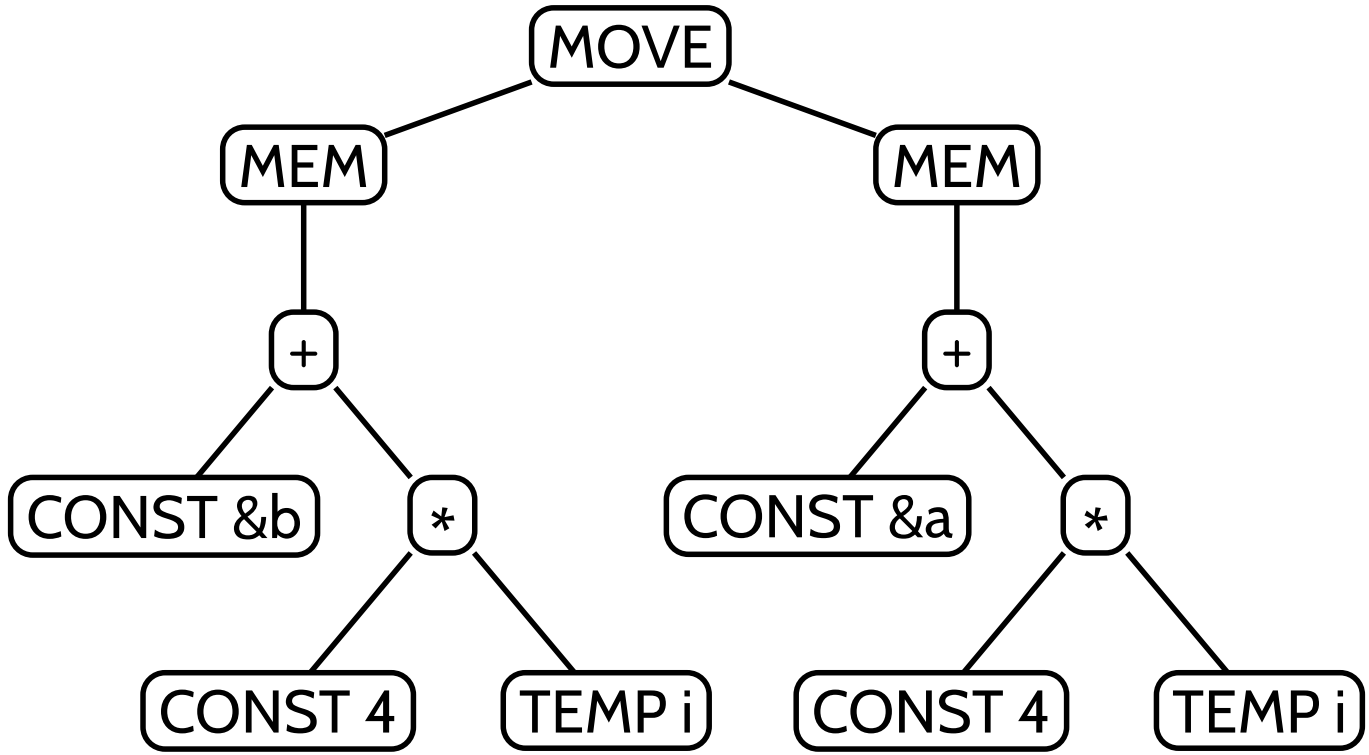
Action:

Running dynamic programming on our IR tree



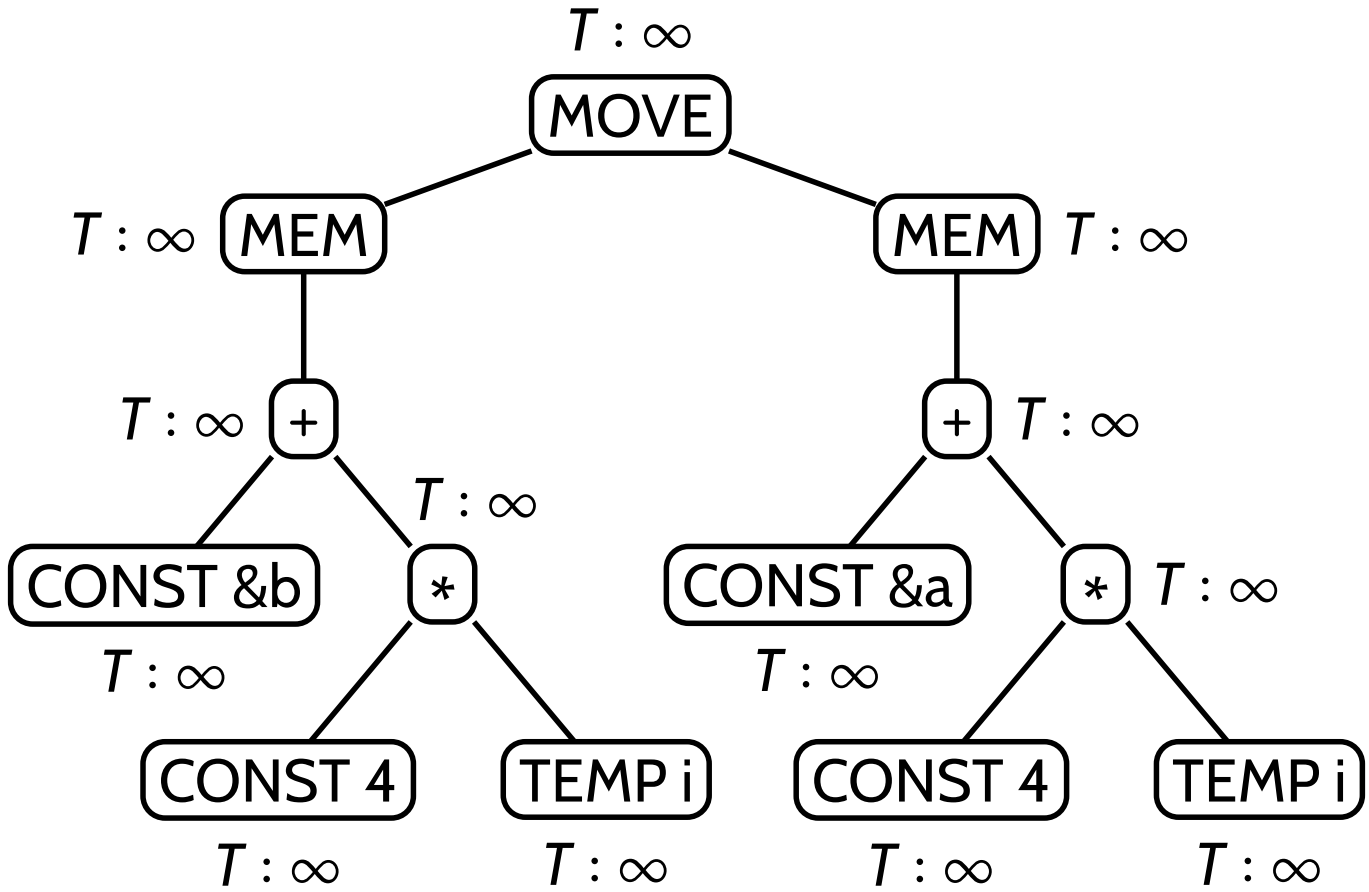
Action: added boxes for better readability

Running dynamic programming on our IR tree



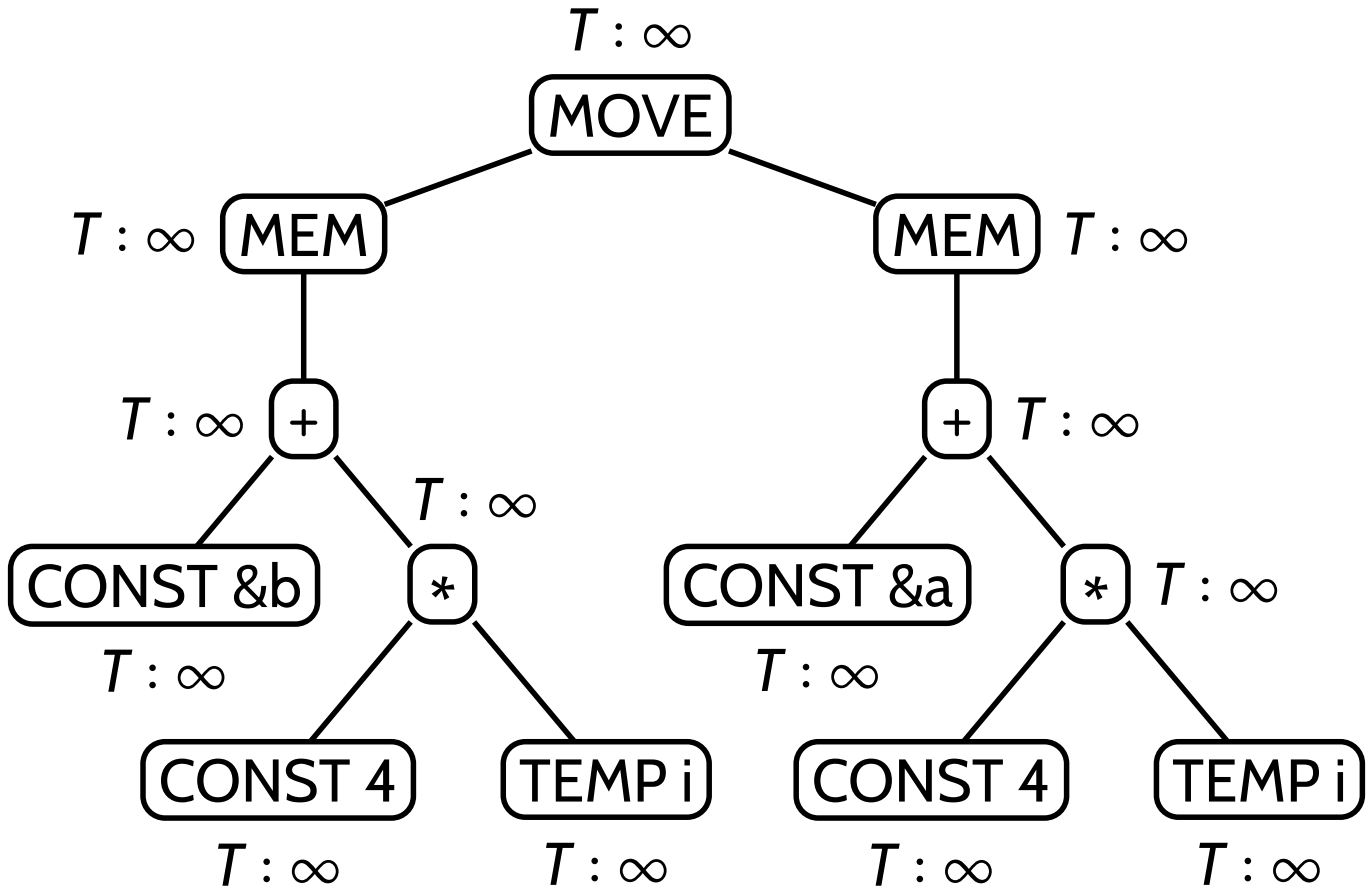
Action: assume tile matches already found

Running dynamic programming on our IR tree



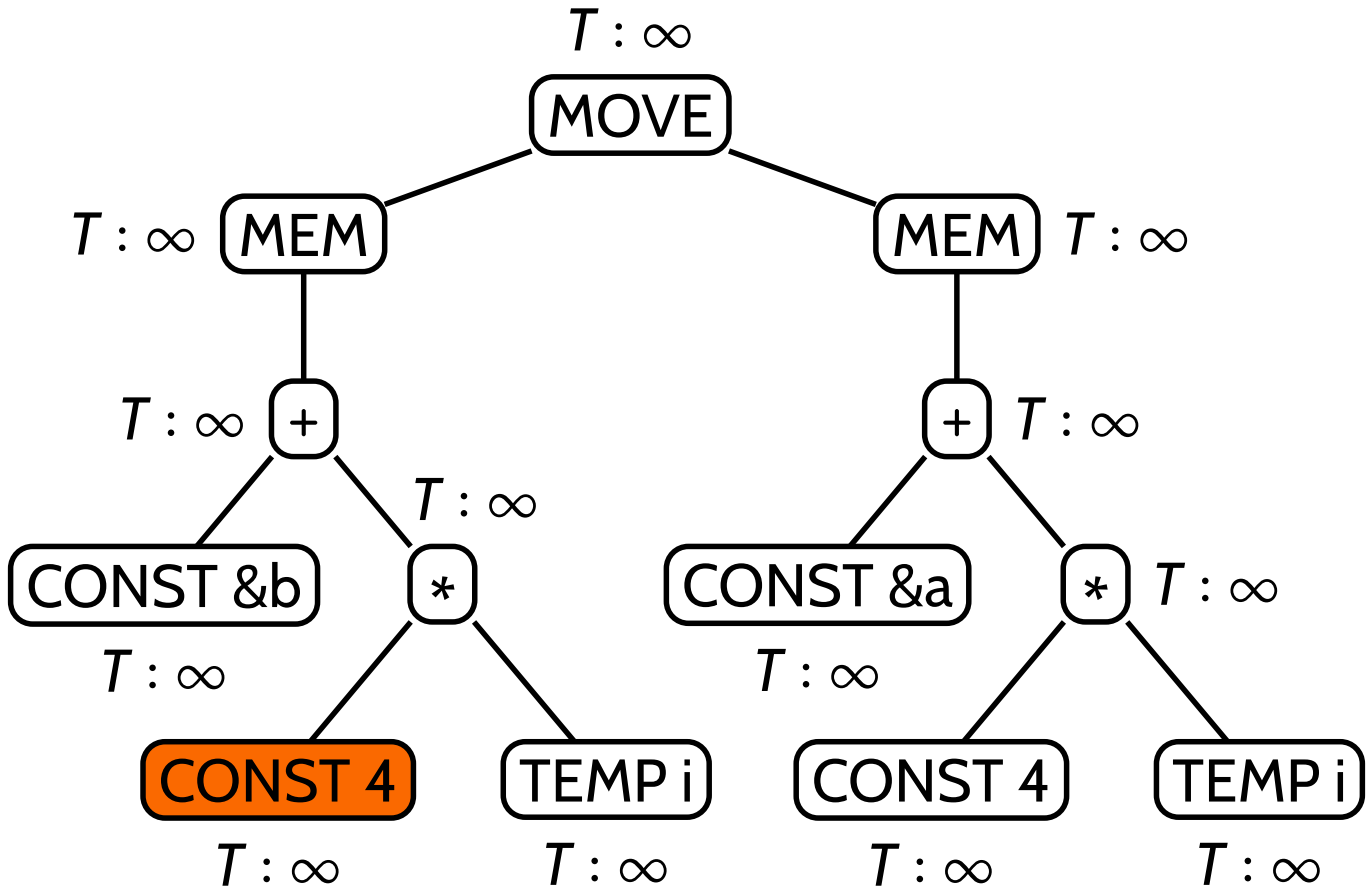
Action: initialize reduction costs

Running dynamic programming on our IR tree



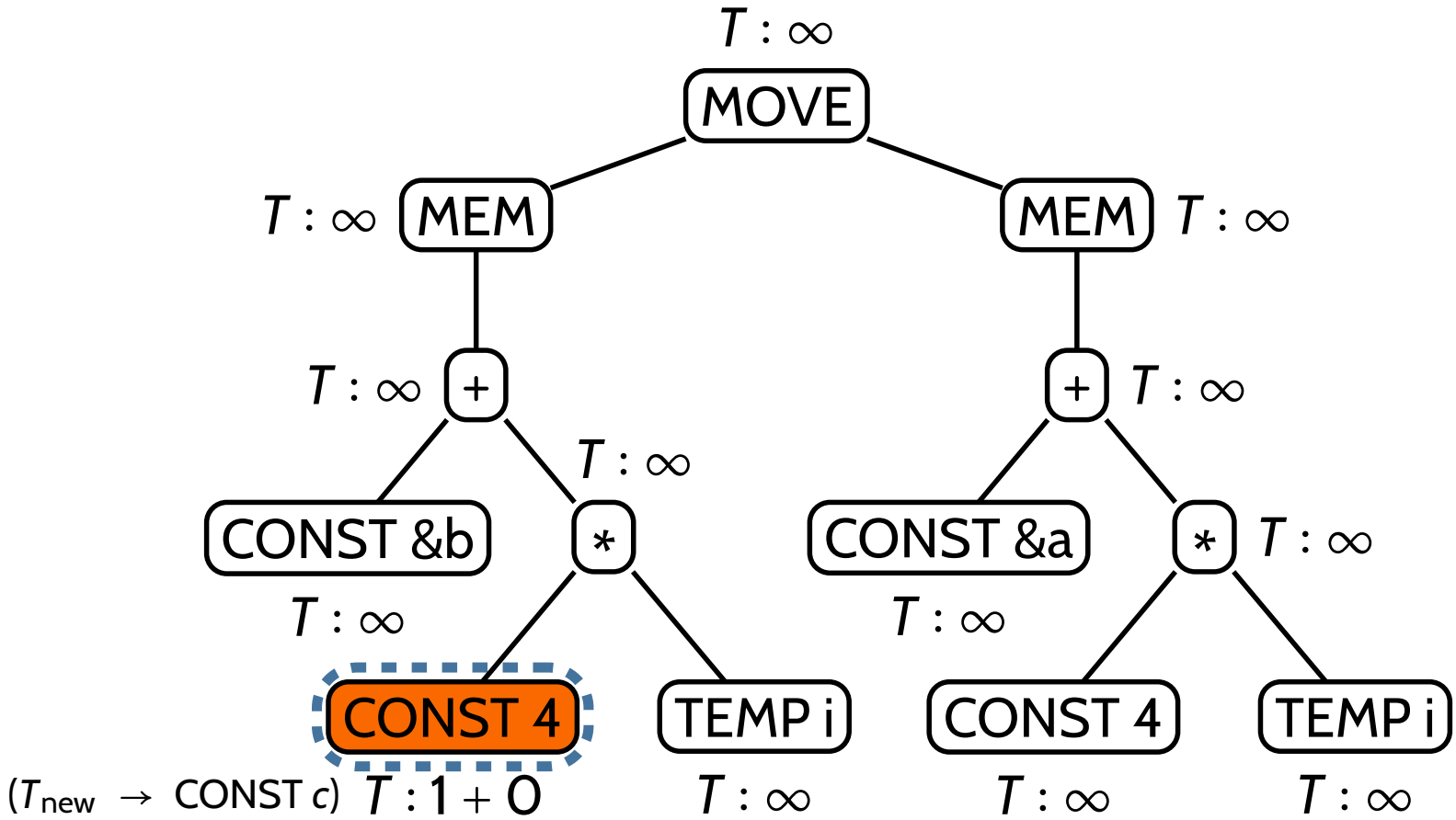
Action: compute least reduction costs

Running dynamic programming on our IR tree



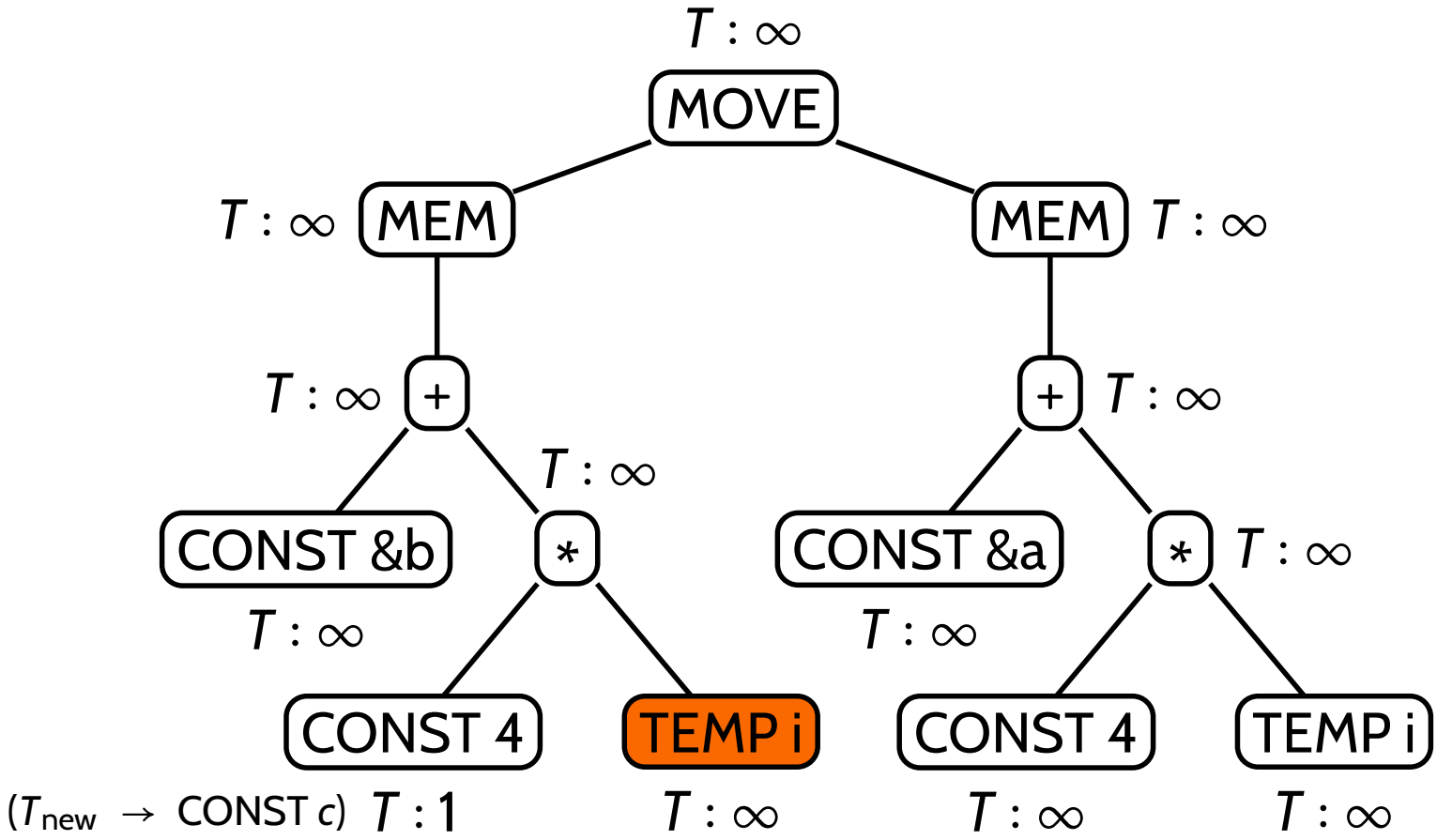
Action: compute least reduction costs

Running dynamic programming on our IR tree



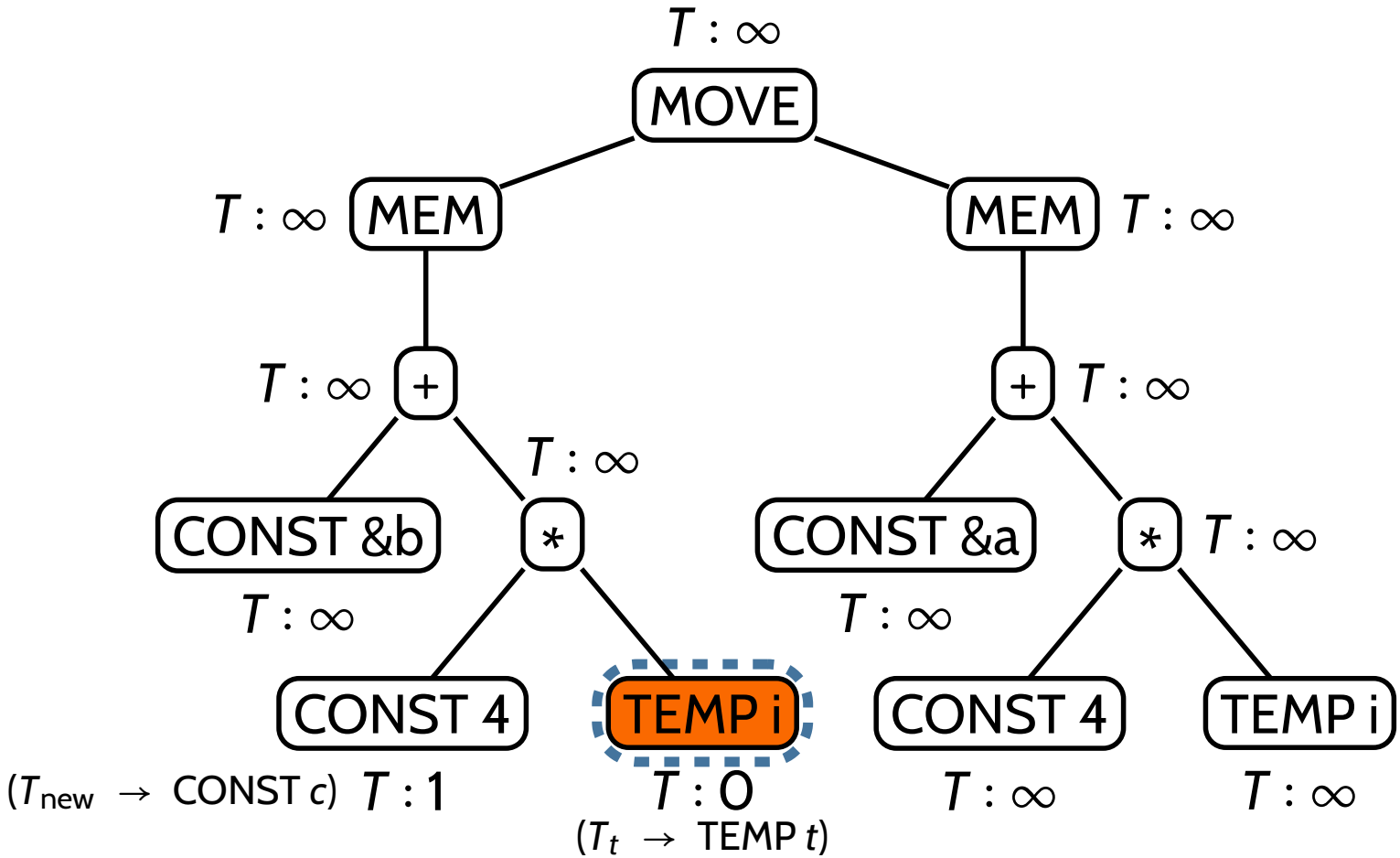
Action: compute least reduction costs

Running dynamic programming on our IR tree



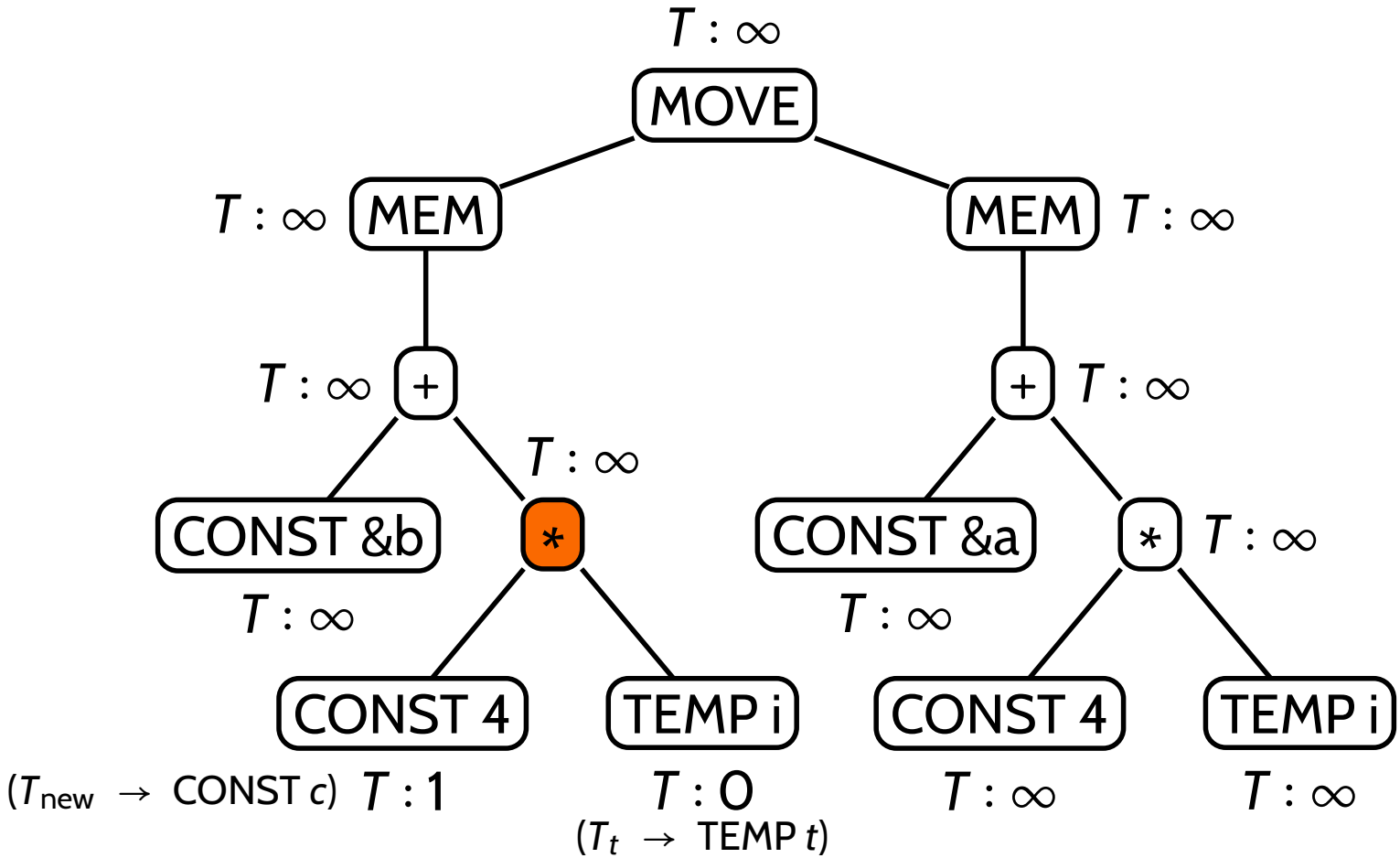
Action: compute least reduction costs

Running dynamic programming on our IR tree



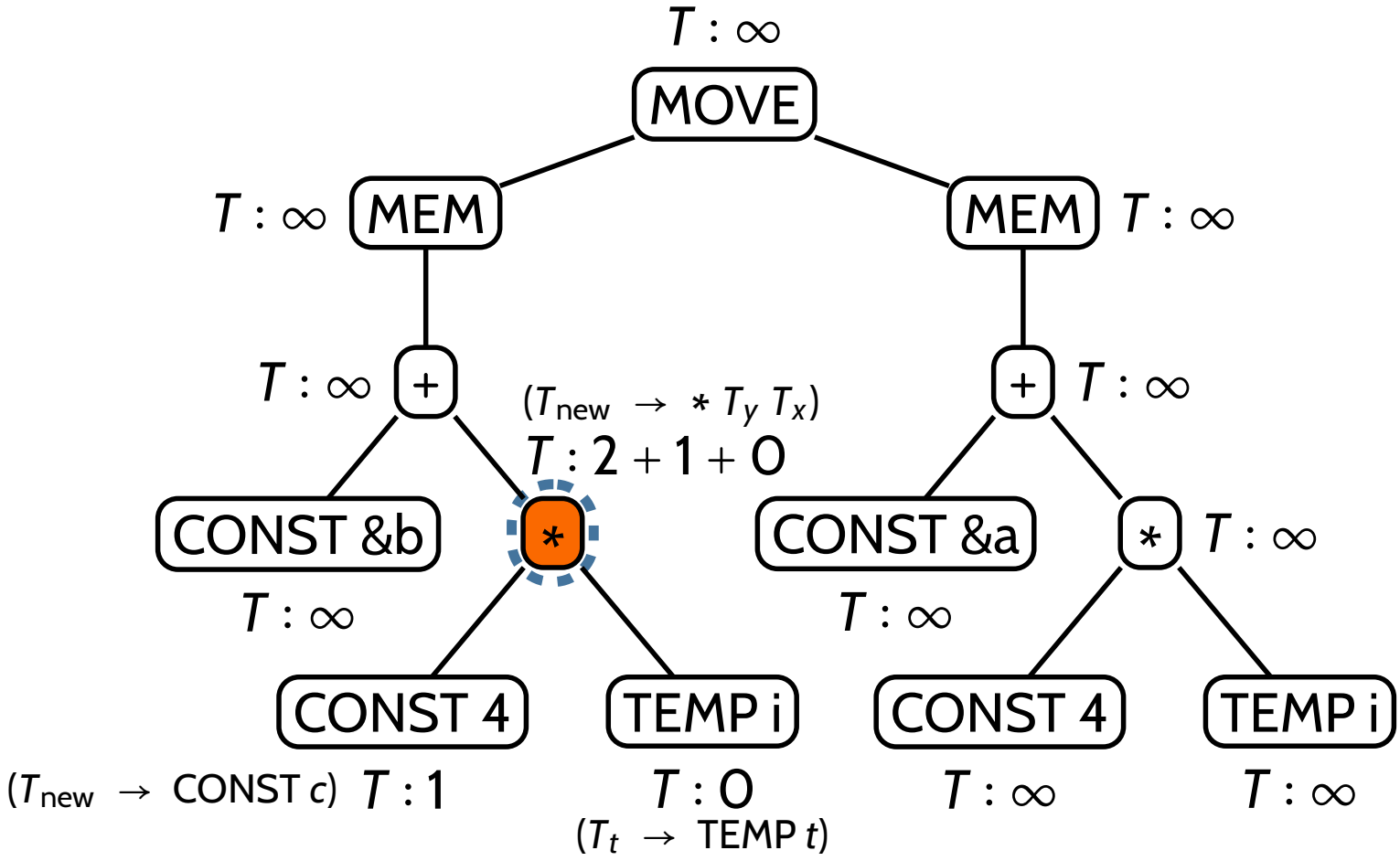
Action: compute least reduction costs

Running dynamic programming on our IR tree



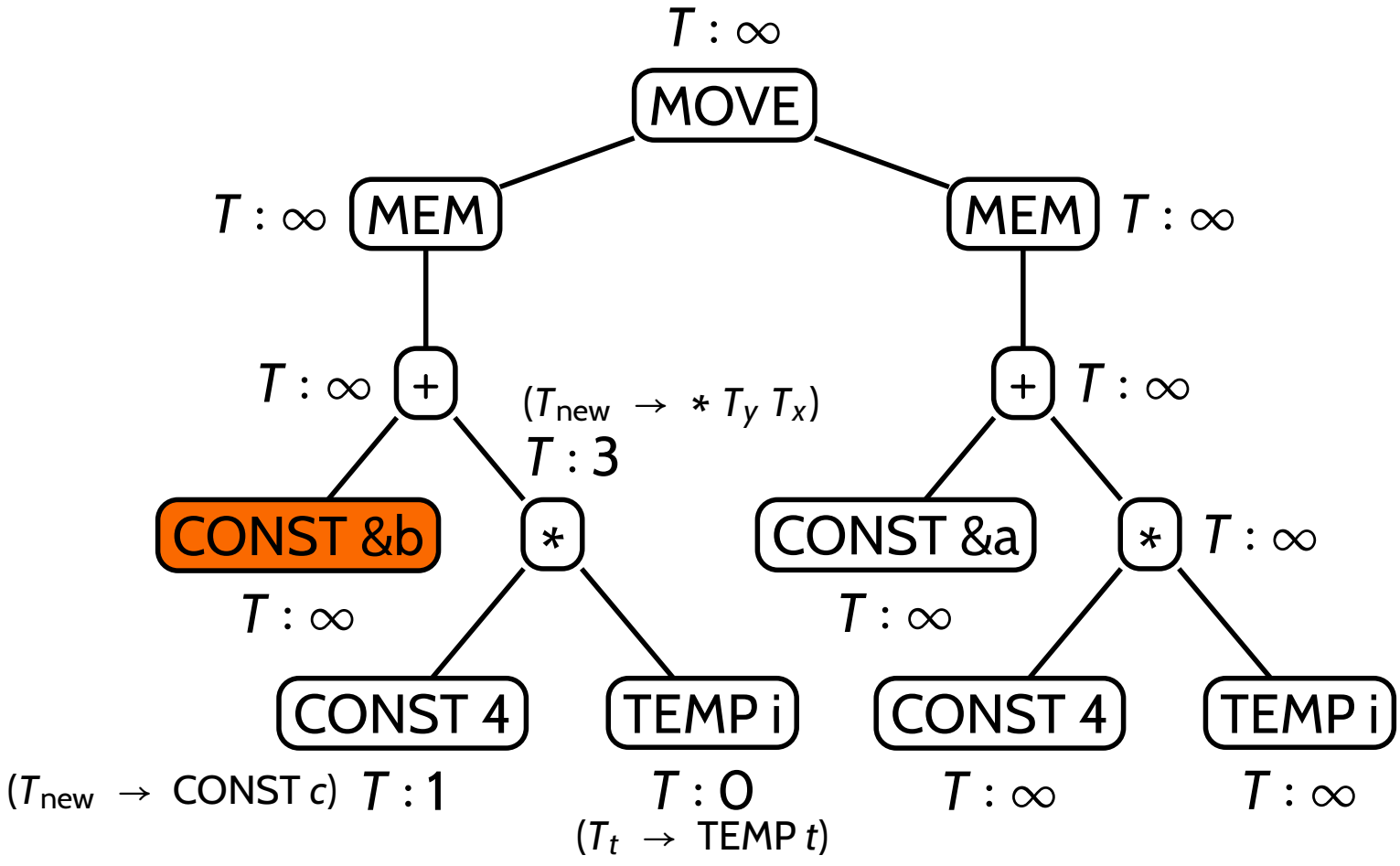
Action: compute least reduction costs

Running dynamic programming on our IR tree



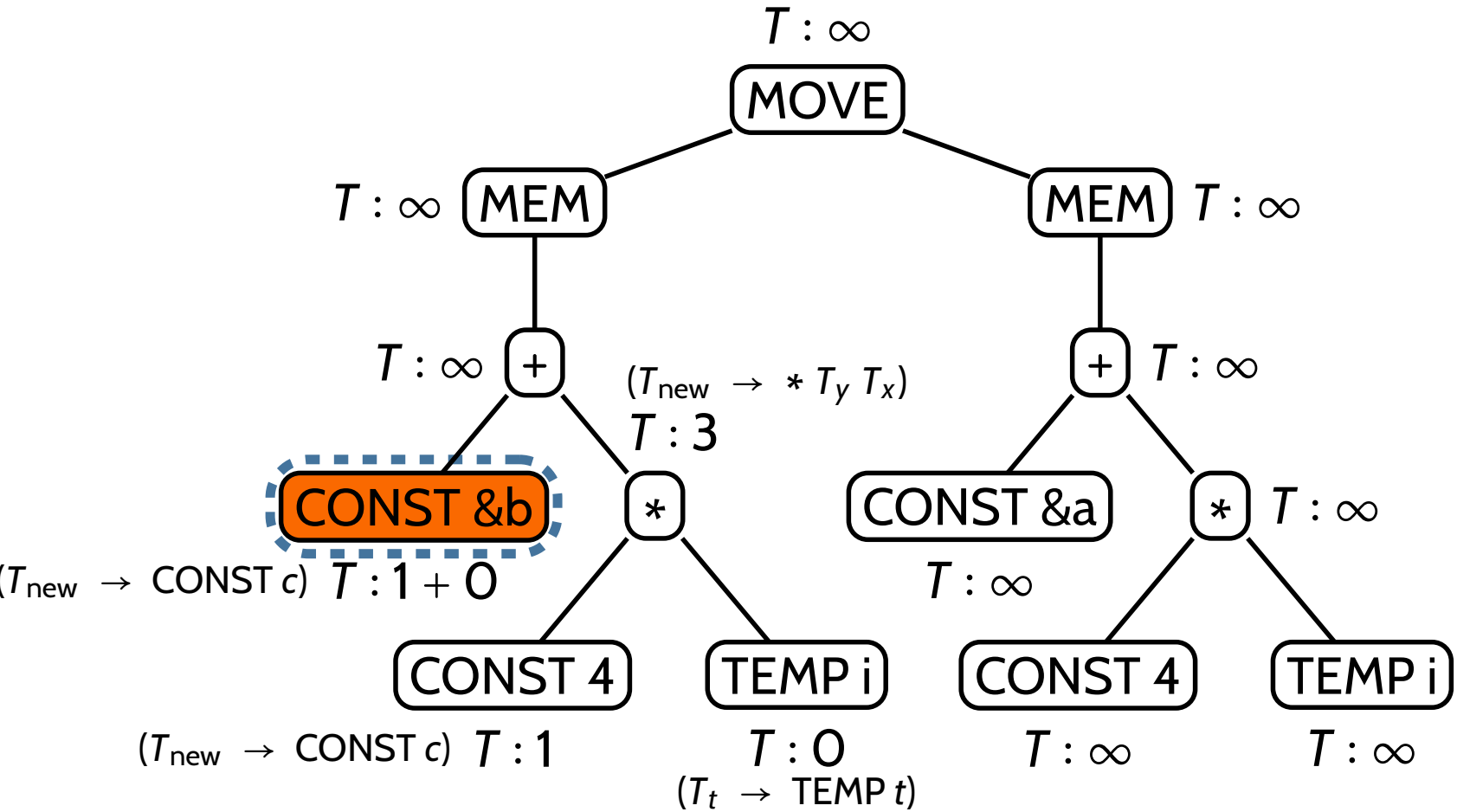
Action: compute least reduction costs

Running dynamic programming on our IR tree



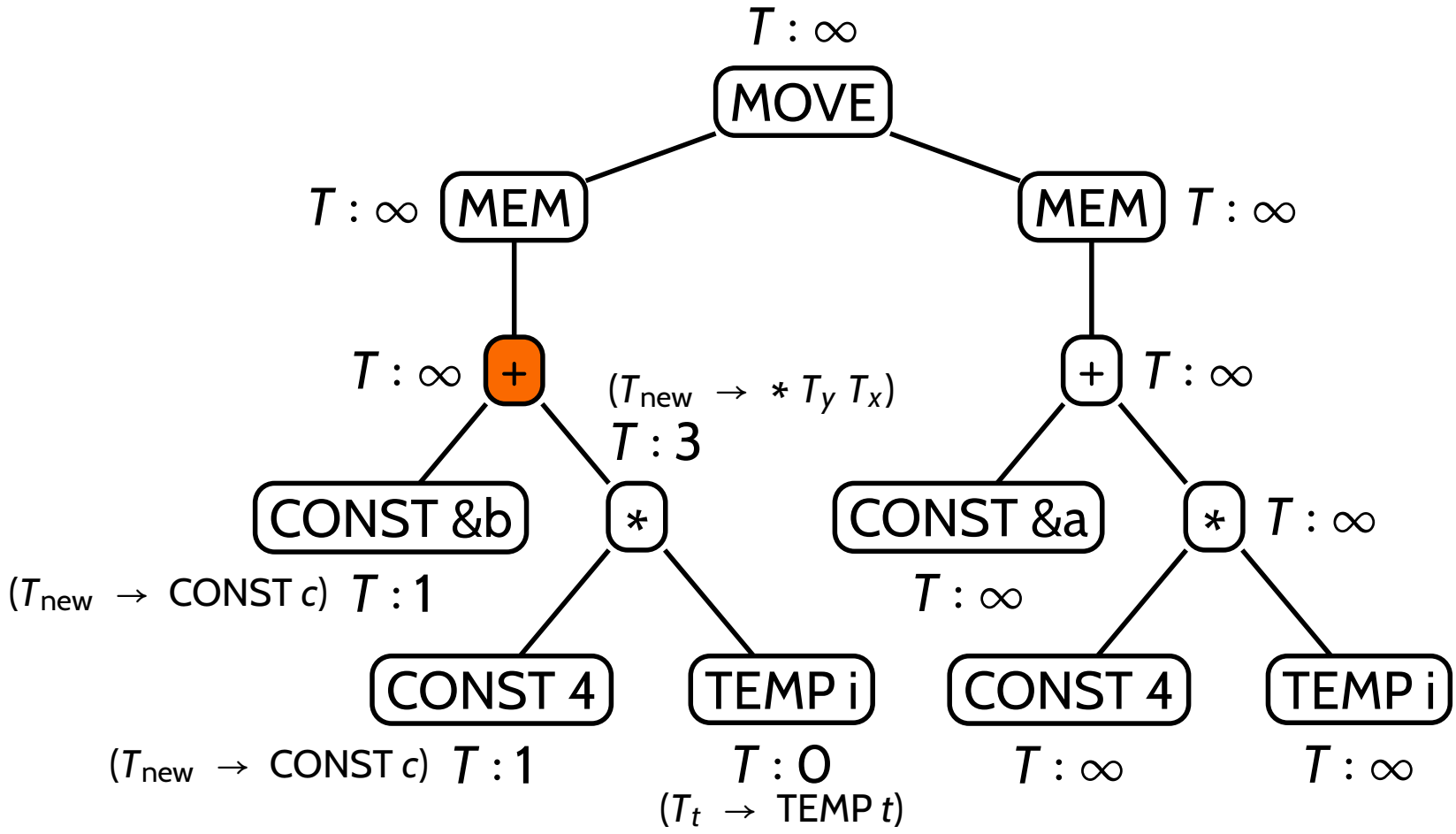
Action: compute least reduction costs

Running dynamic programming on our IR tree



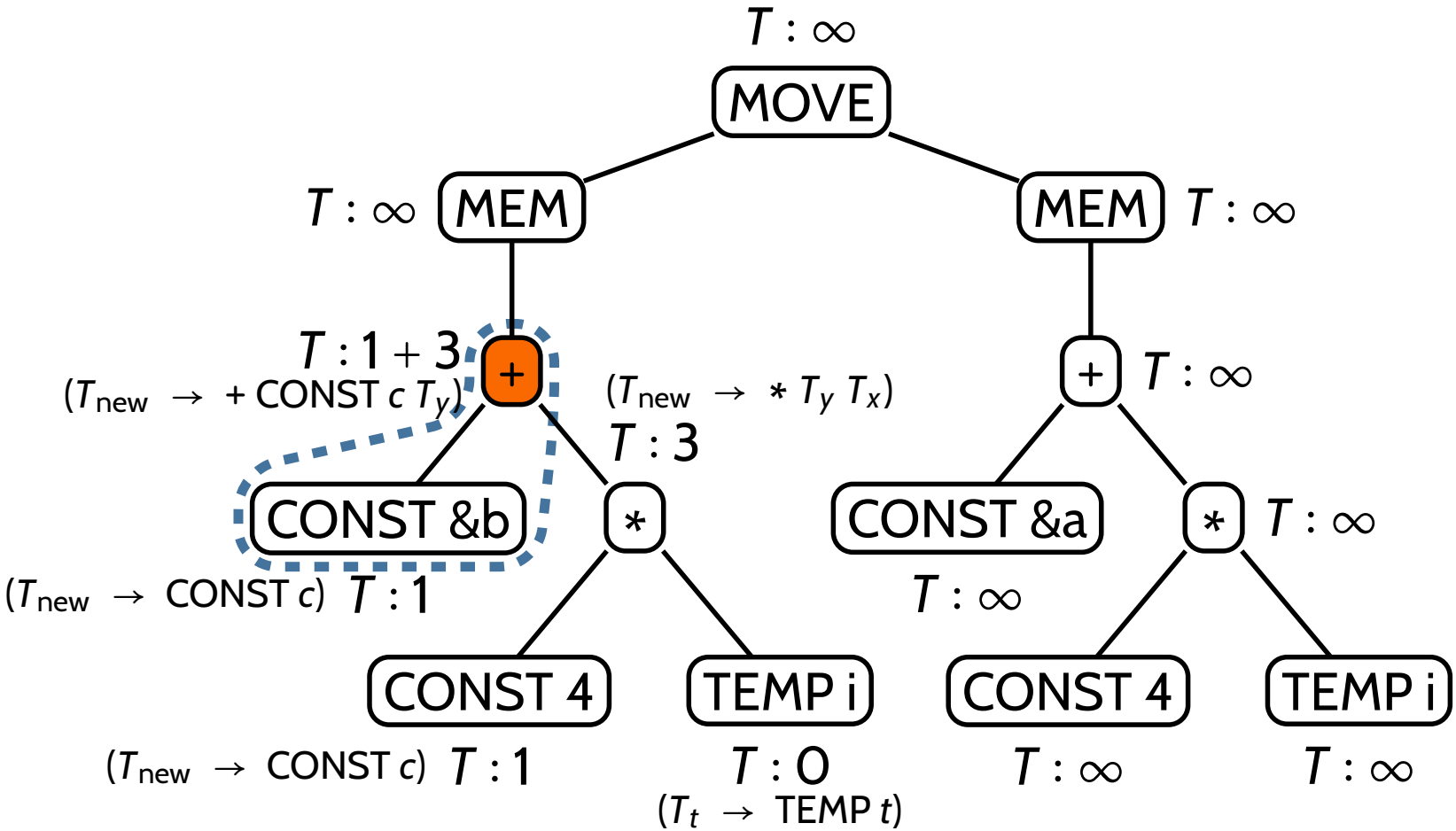
Action: compute least reduction costs

Running dynamic programming on our IR tree



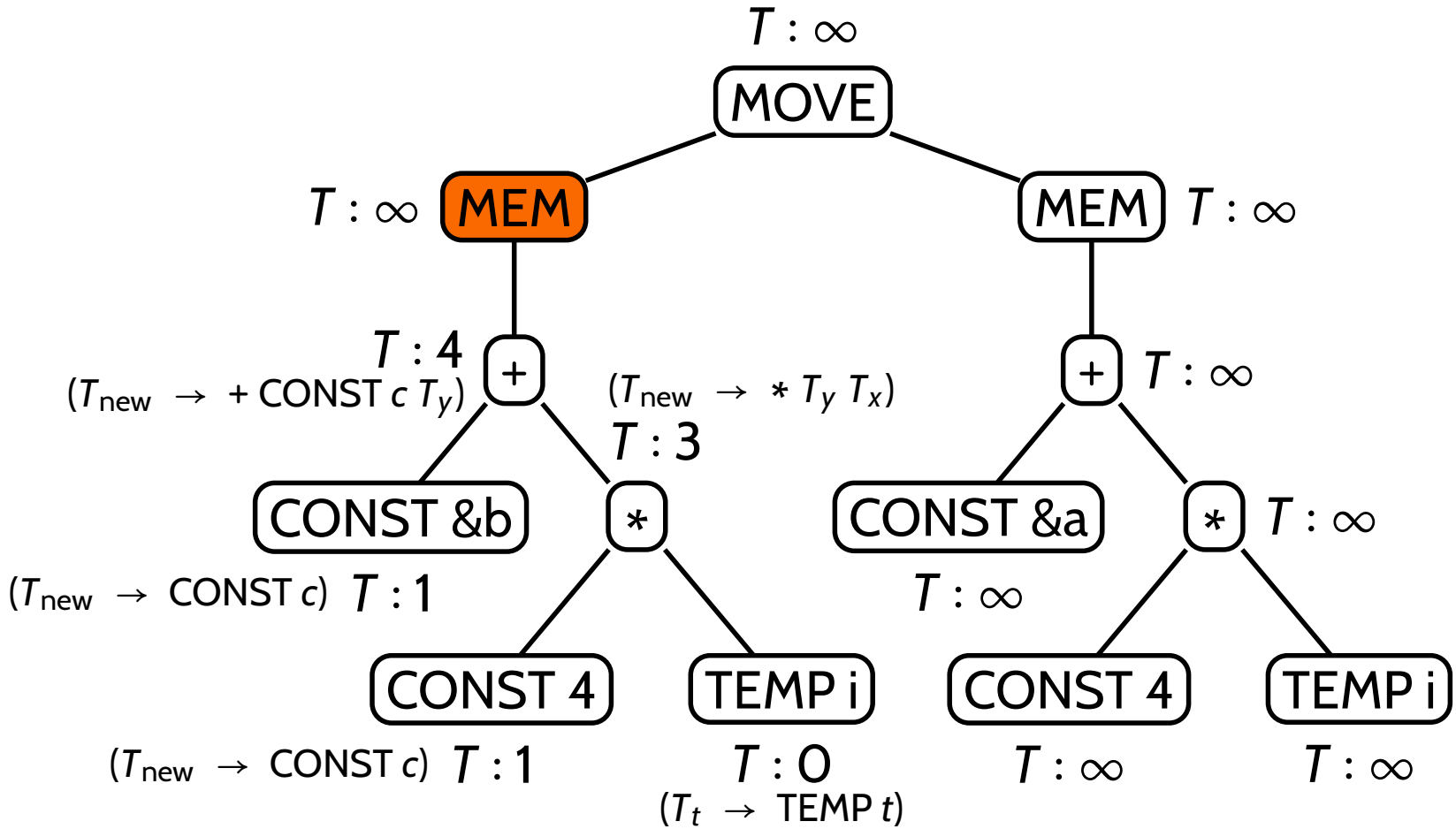
Action: compute least reduction costs

Running dynamic programming on our IR tree



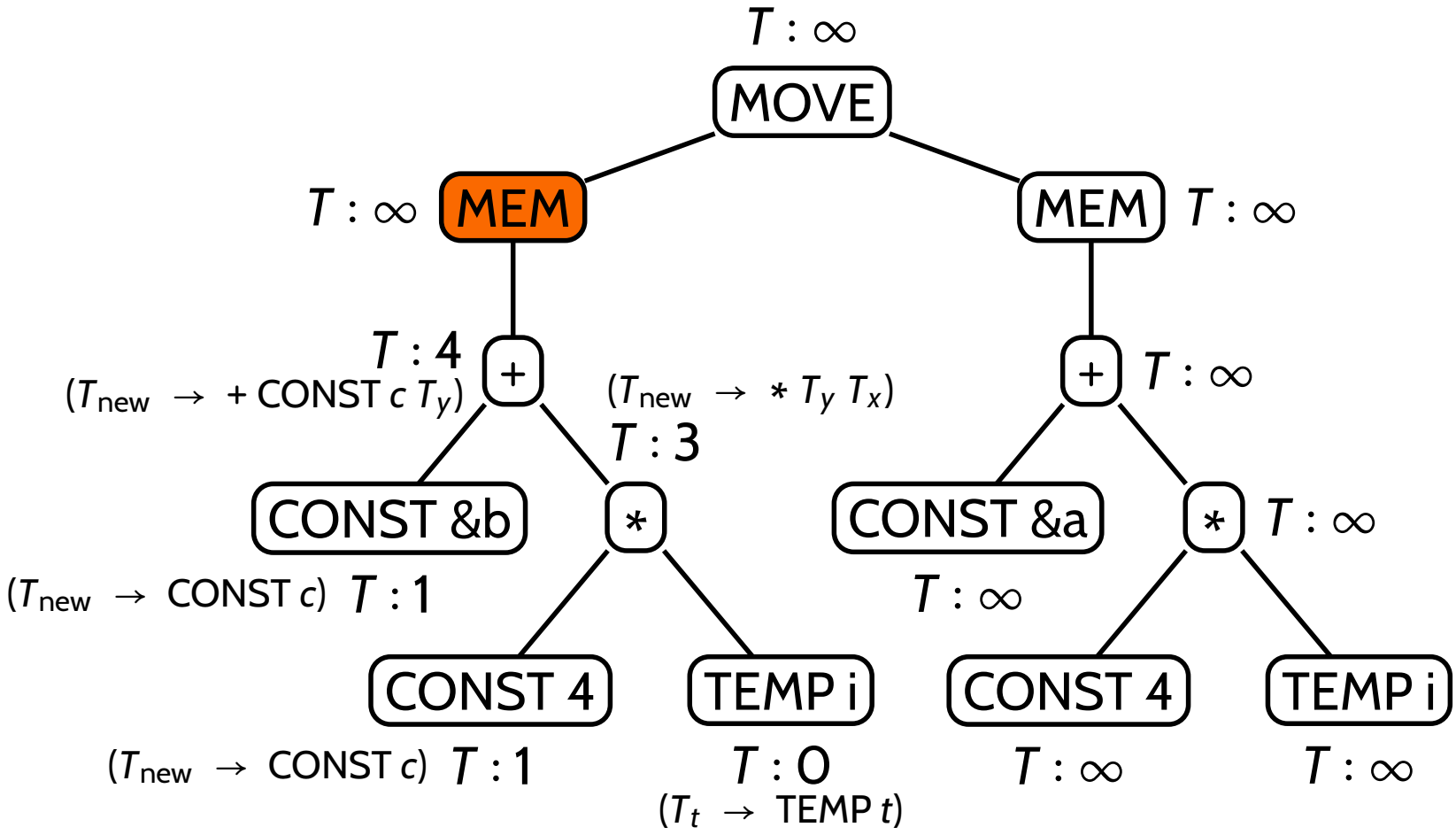
Action: compute least reduction costs

Running dynamic programming on our IR tree



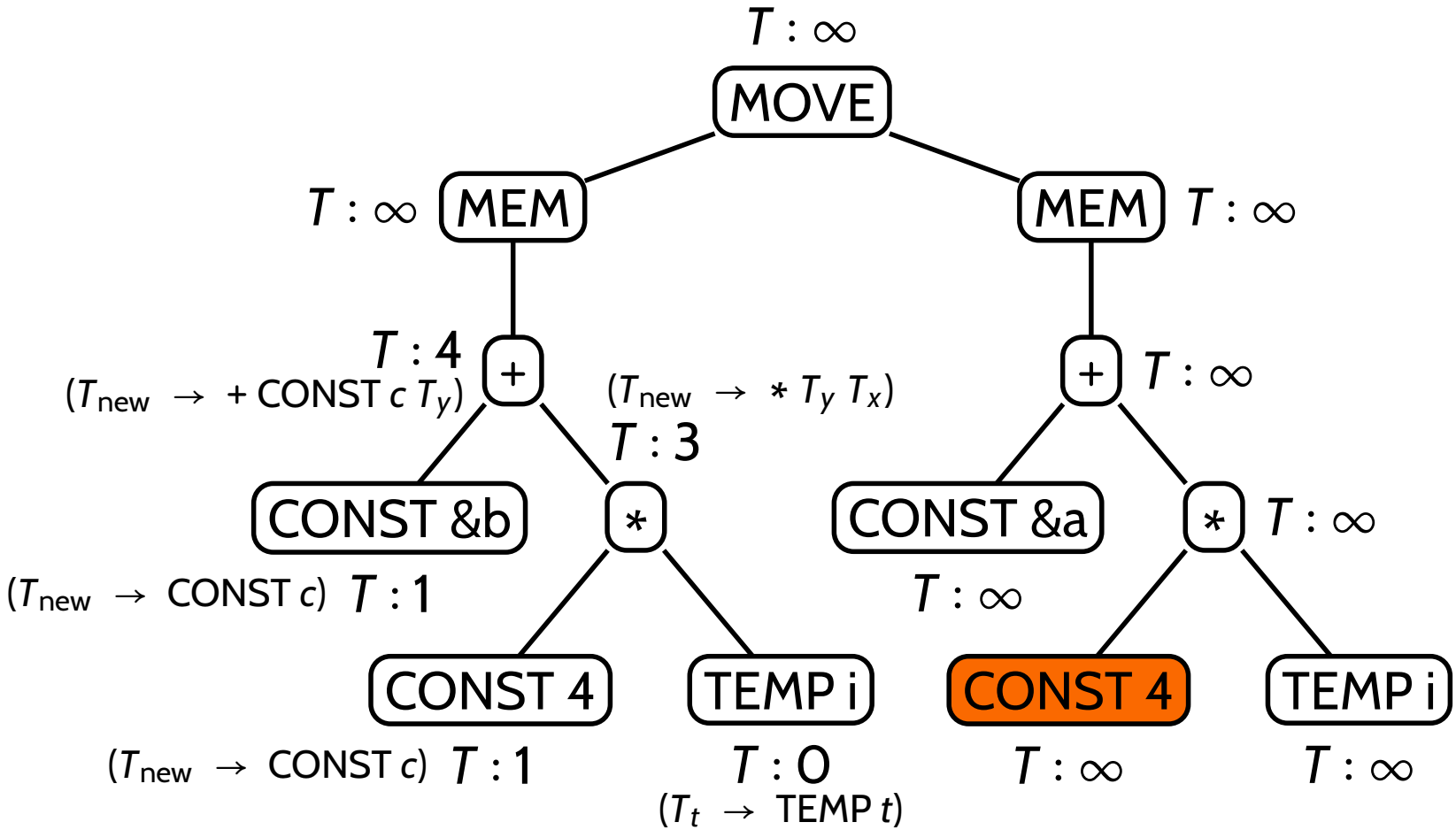
Action: compute least reduction costs

Running dynamic programming on our IR tree



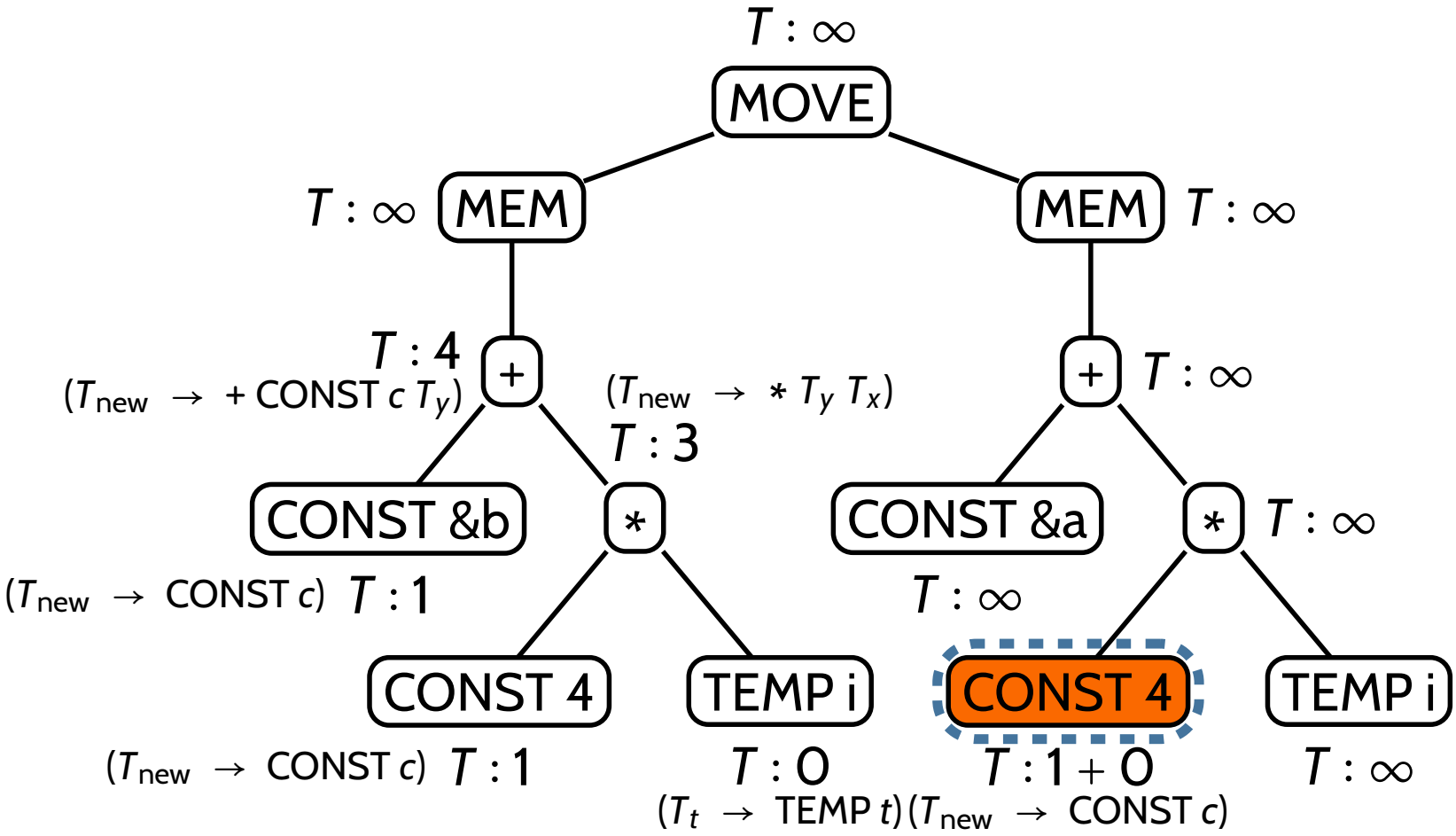
Action: LOAD instructions not allowed here (l-value)

Running dynamic programming on our IR tree



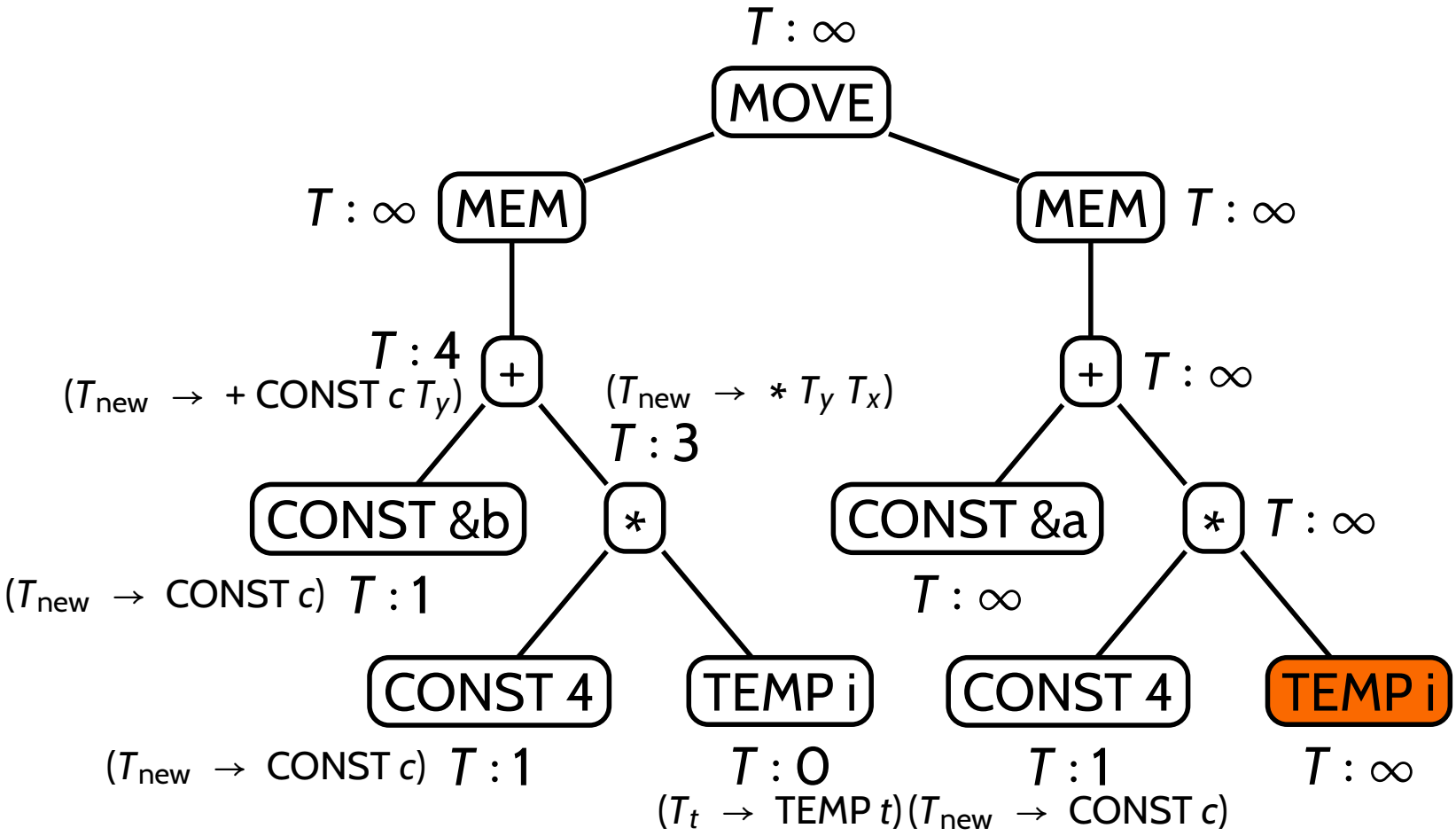
Action: compute least reduction costs

Running dynamic programming on our IR tree



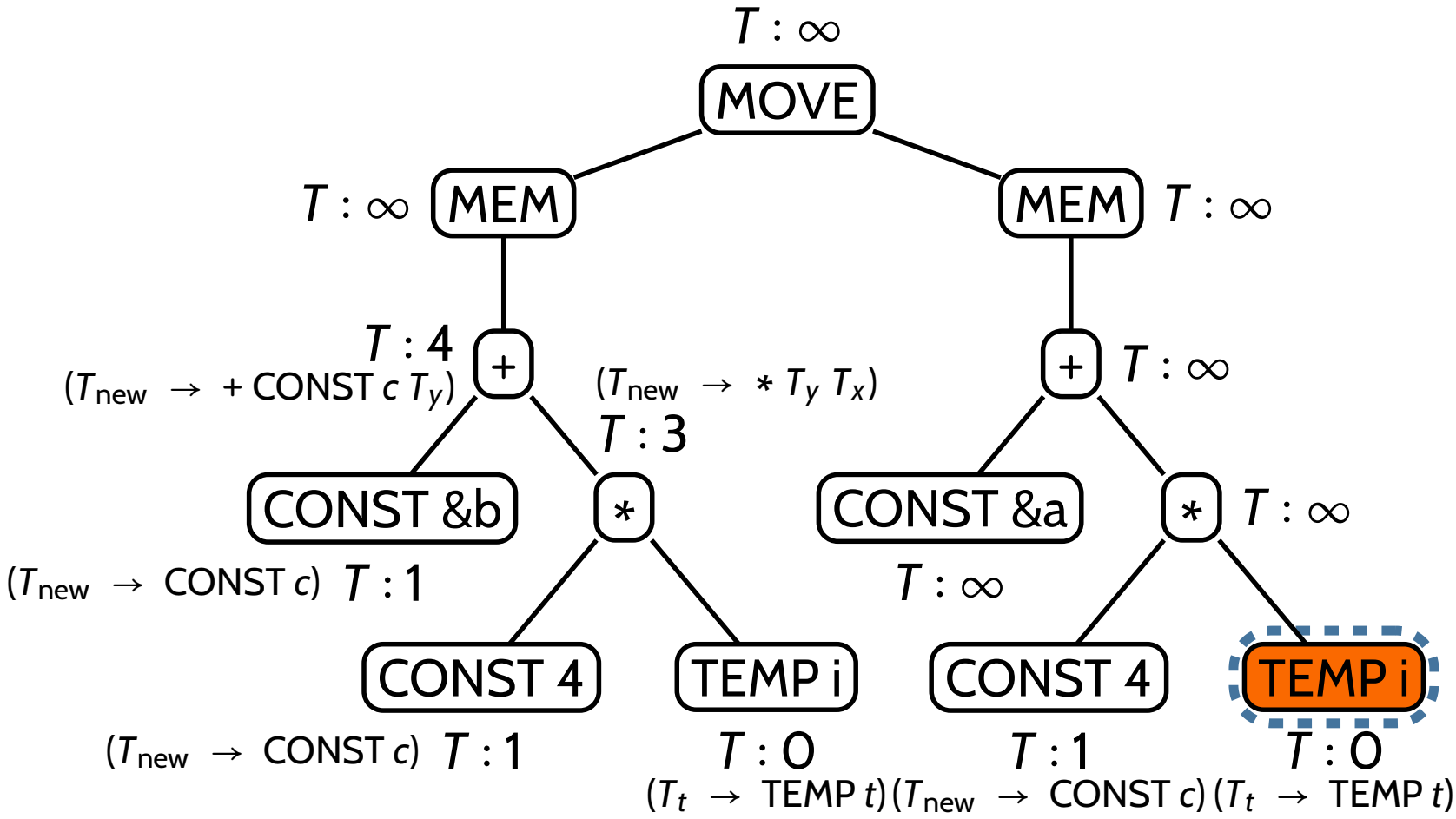
Action: compute least reduction costs

Running dynamic programming on our IR tree



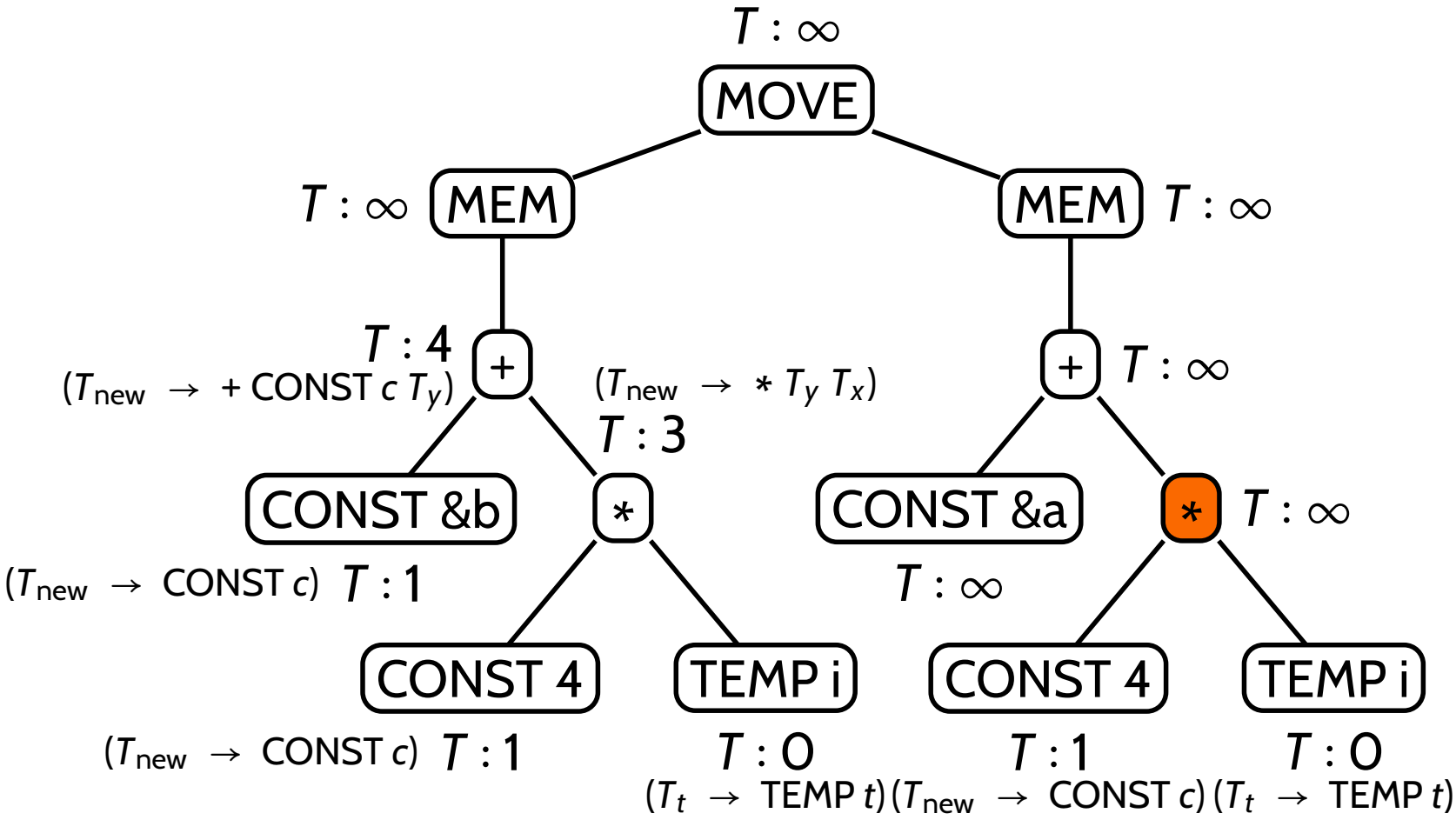
Action: compute least reduction costs

Running dynamic programming on our IR tree



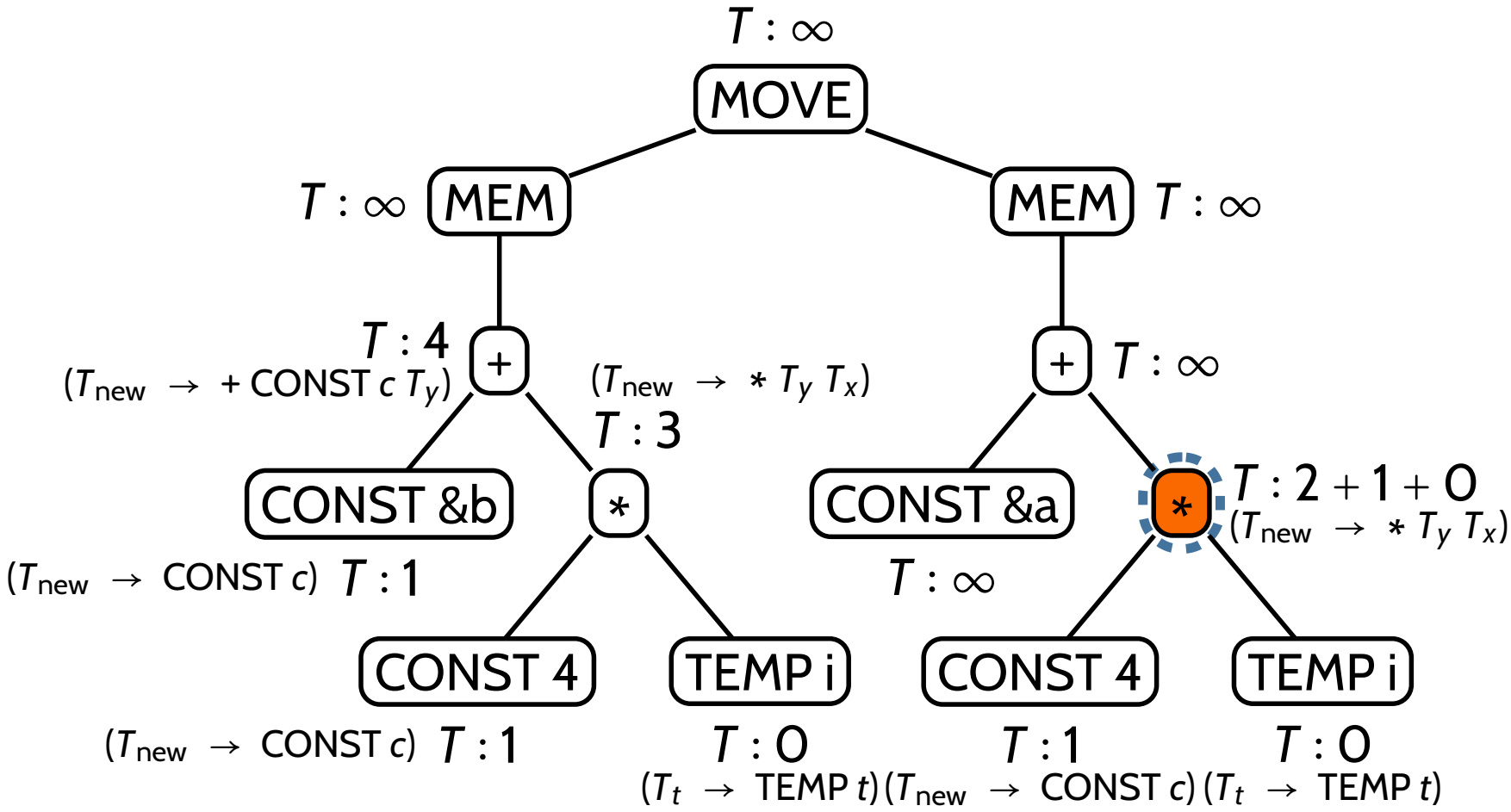
Action: compute least reduction costs

Running dynamic programming on our IR tree



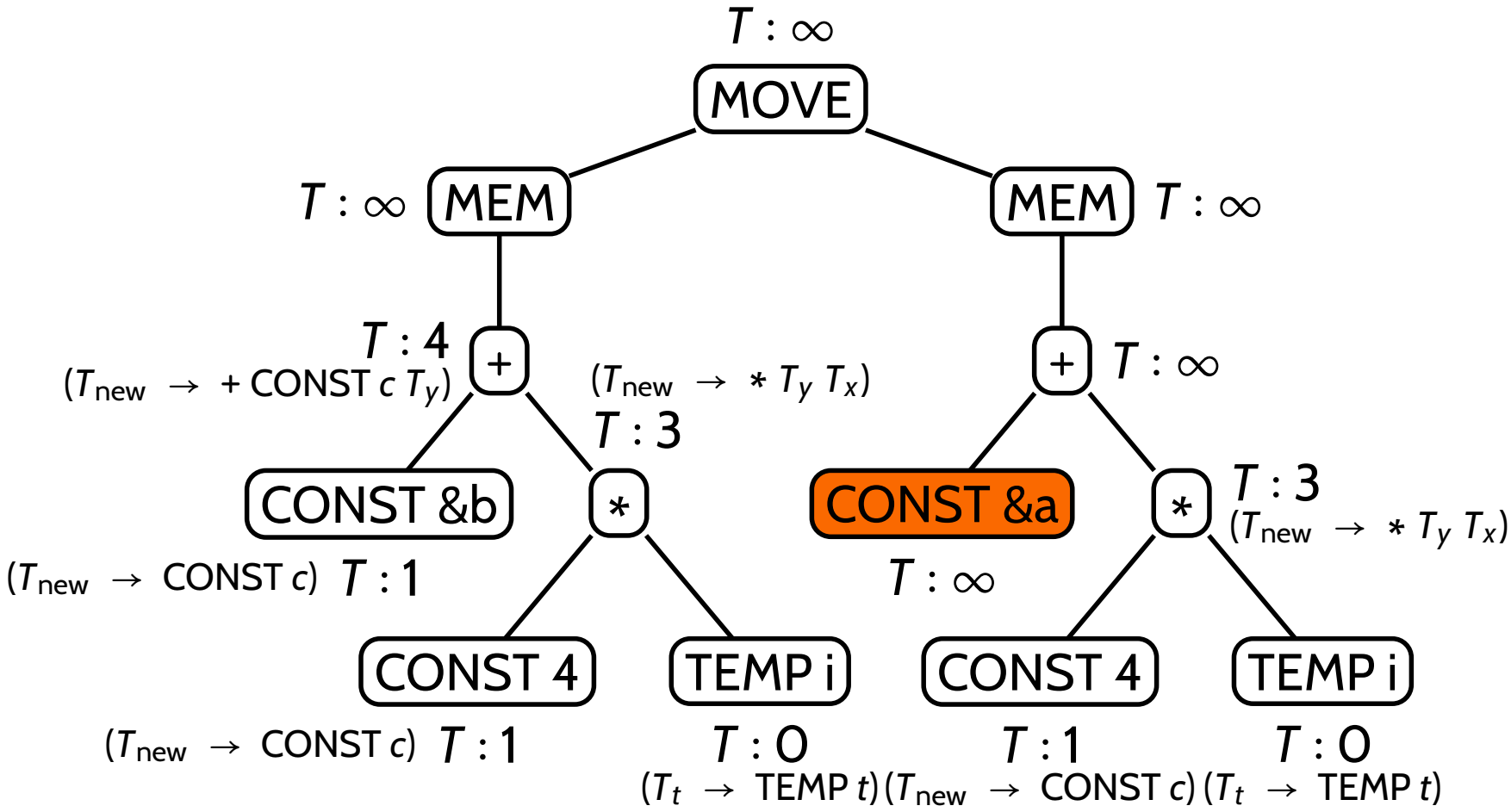
Action: compute least reduction costs

Running dynamic programming on our IR tree



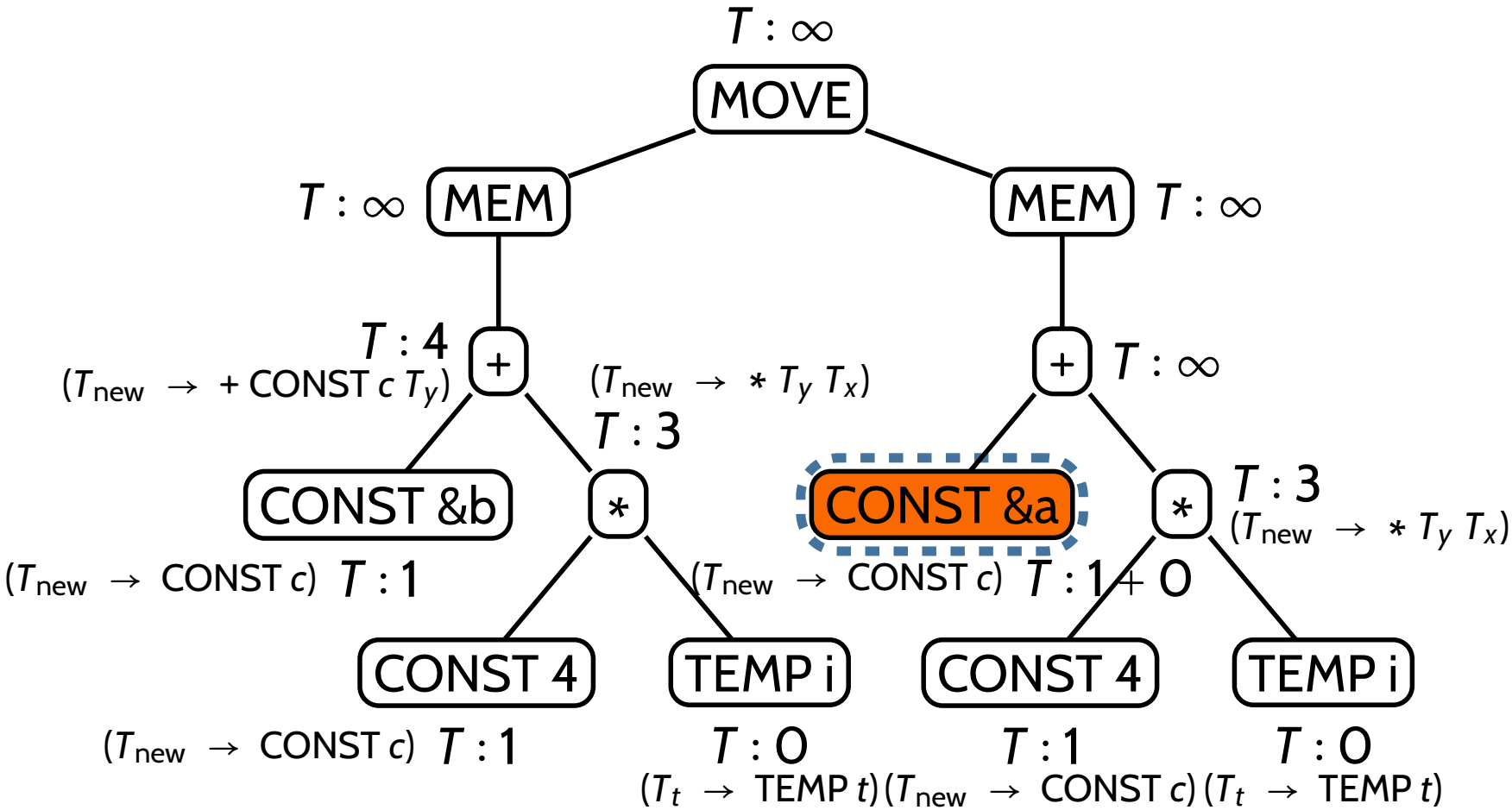
Action: compute least reduction costs

Running dynamic programming on our IR tree



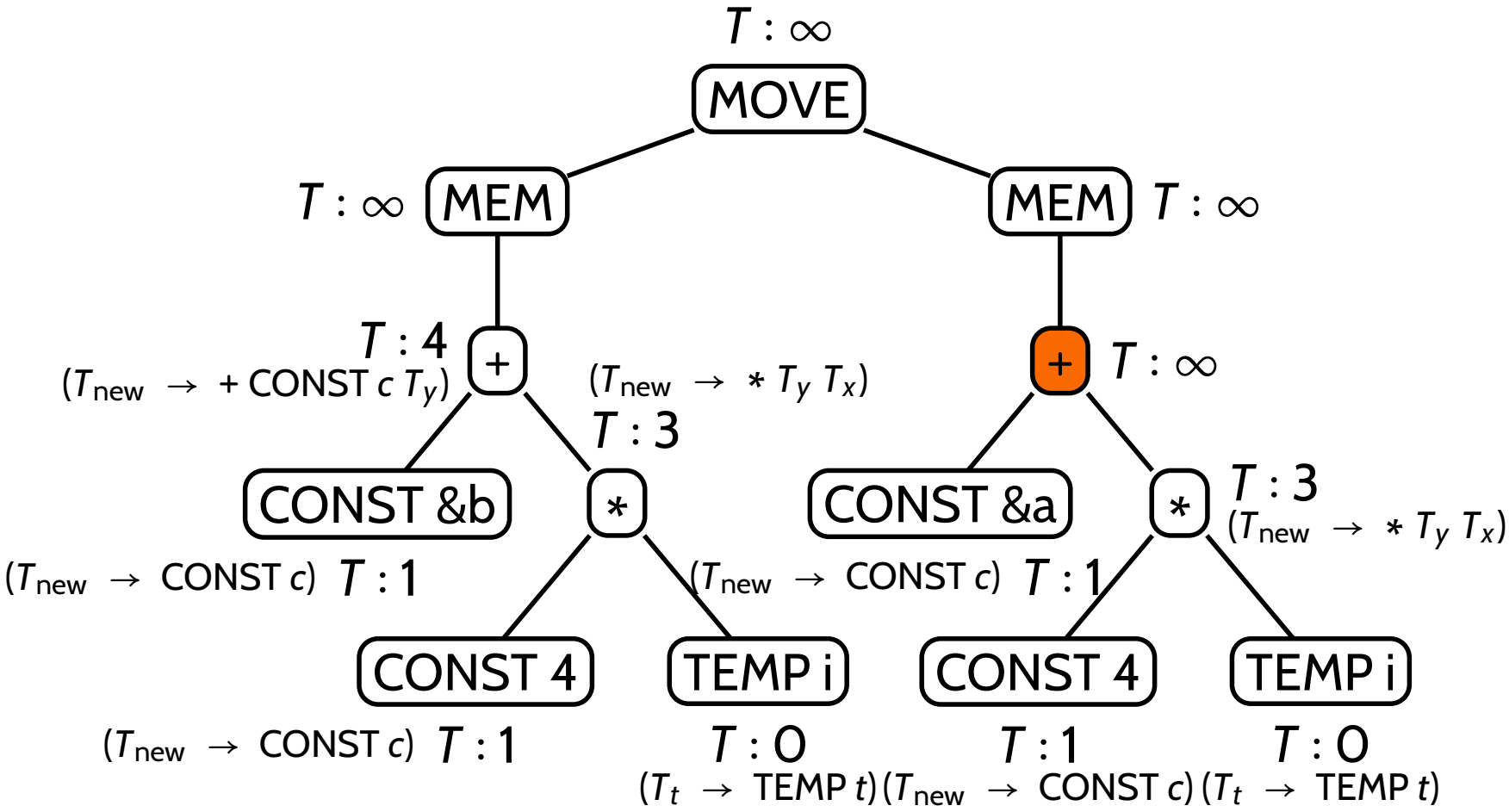
Action: compute least reduction costs

Running dynamic programming on our IR tree



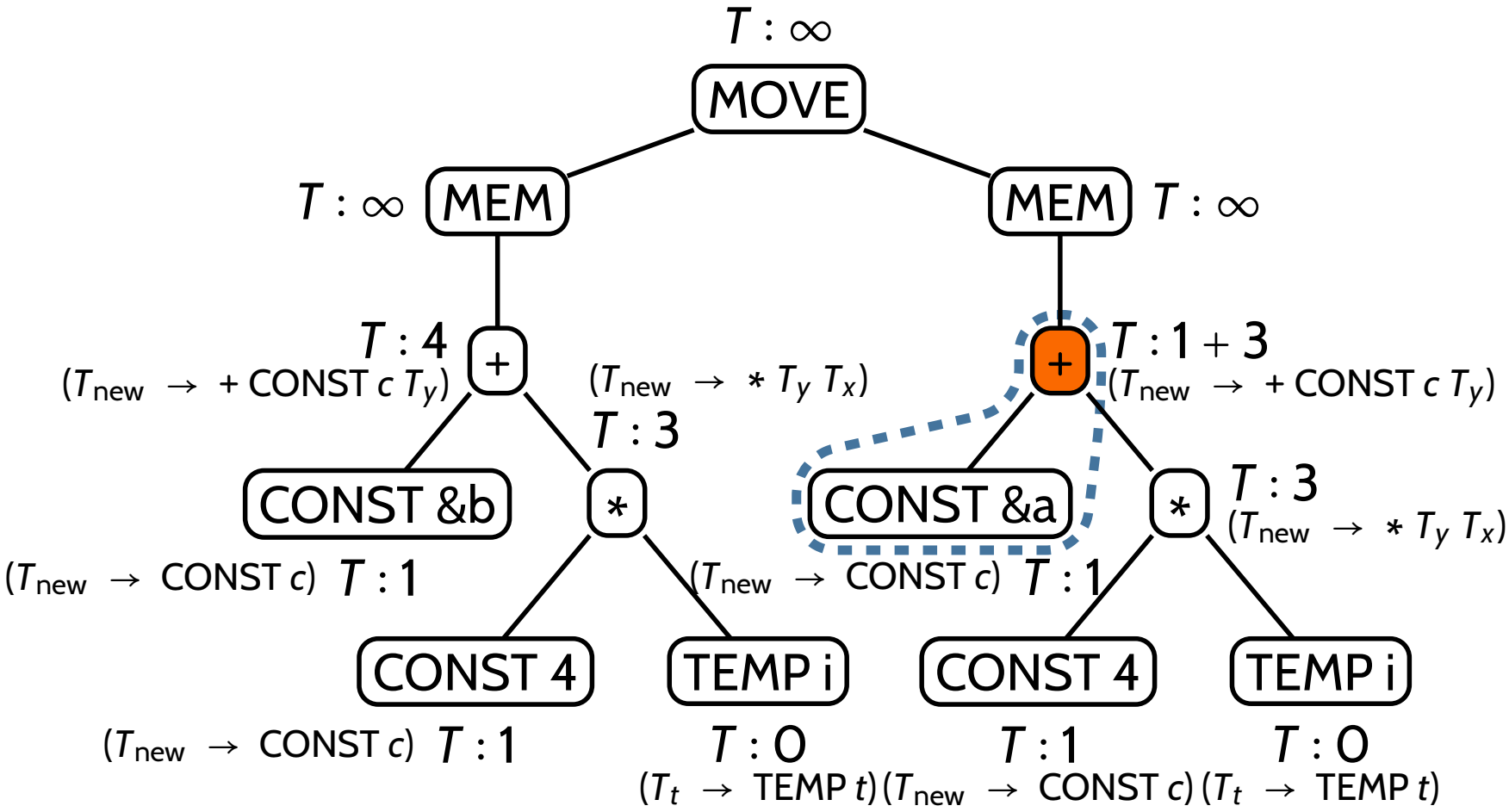
Action: compute least reduction costs

Running dynamic programming on our IR tree



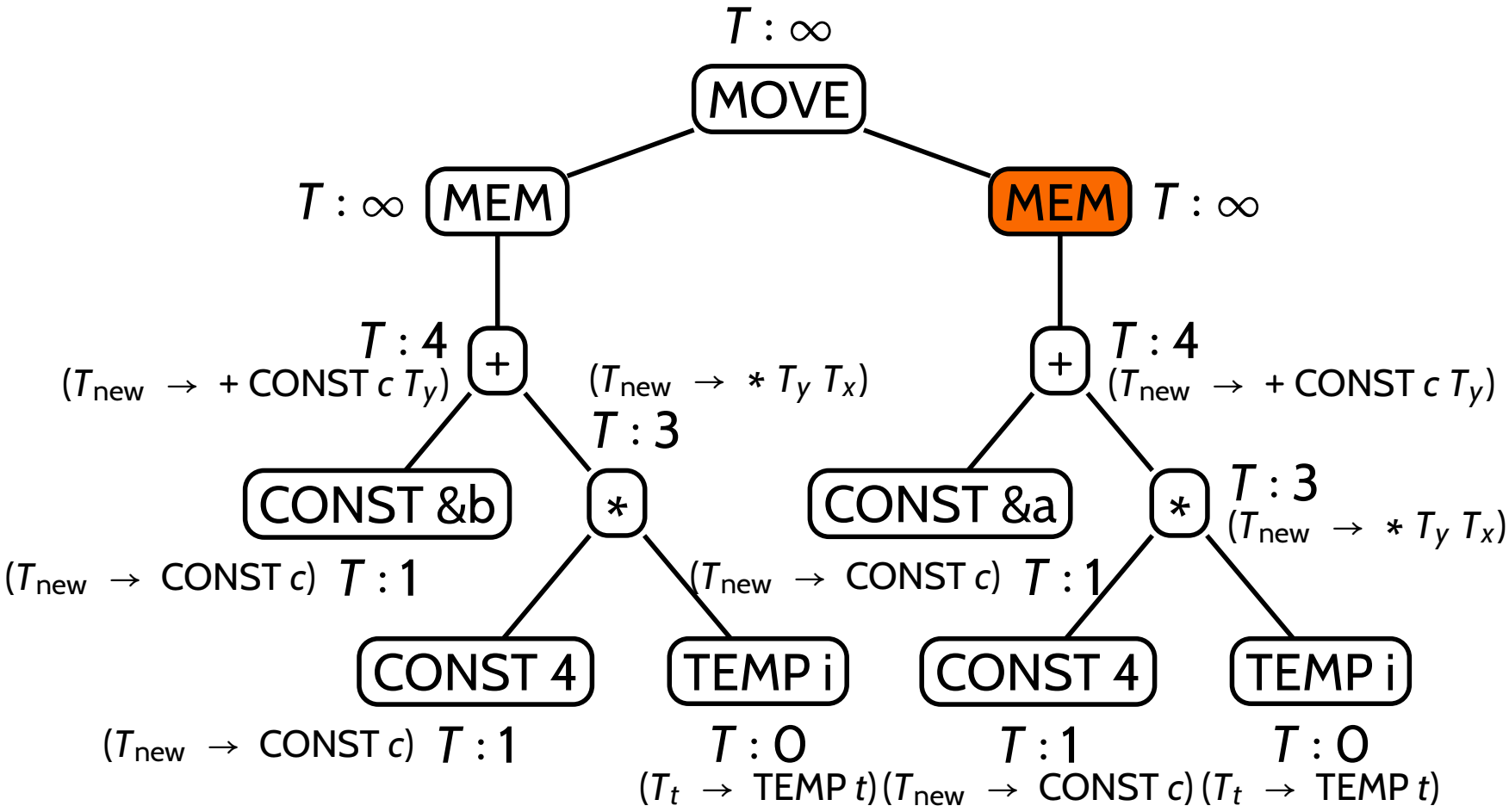
Action: compute least reduction costs

Running dynamic programming on our IR tree



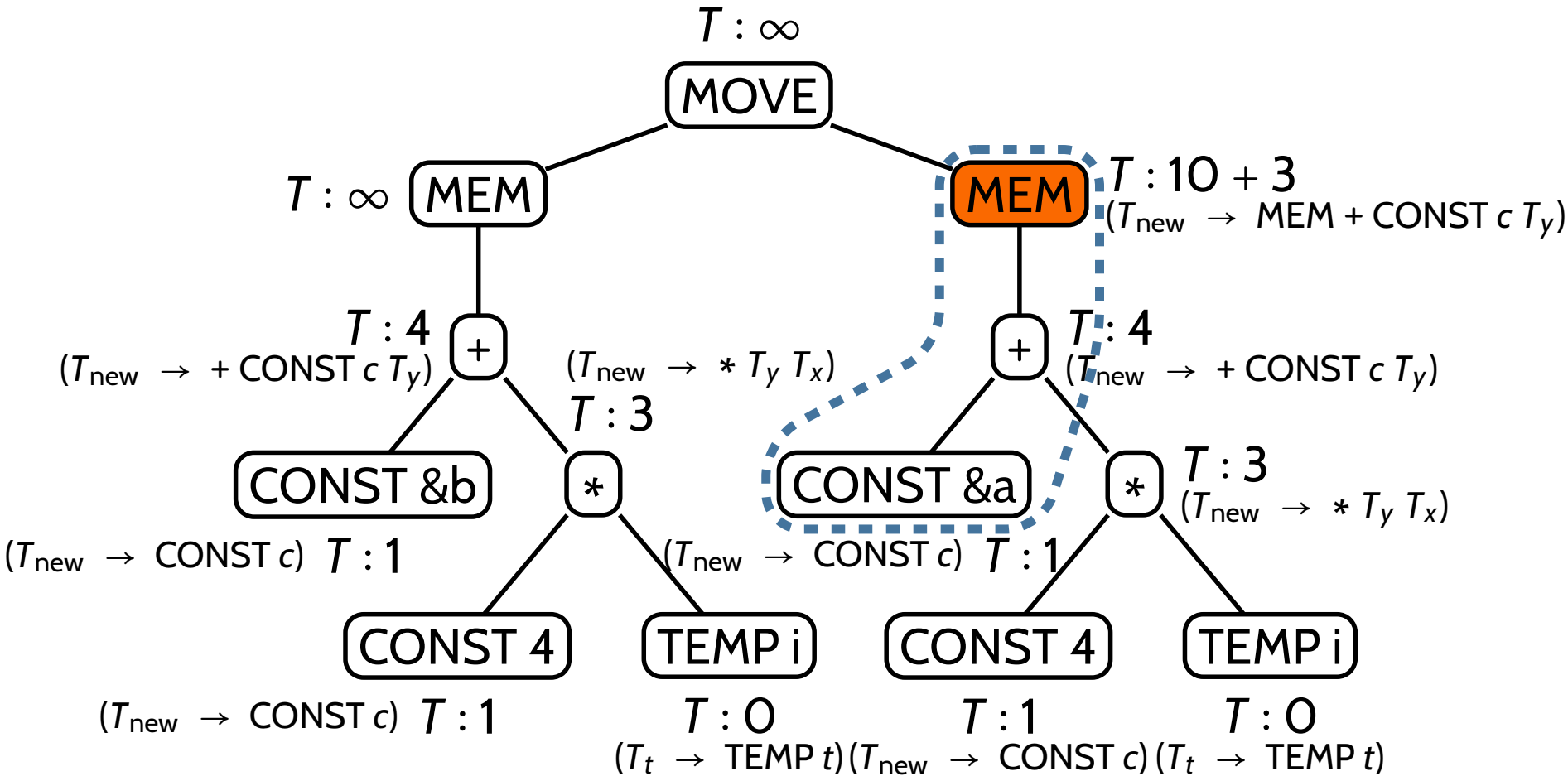
Action: compute least reduction costs

Running dynamic programming on our IR tree



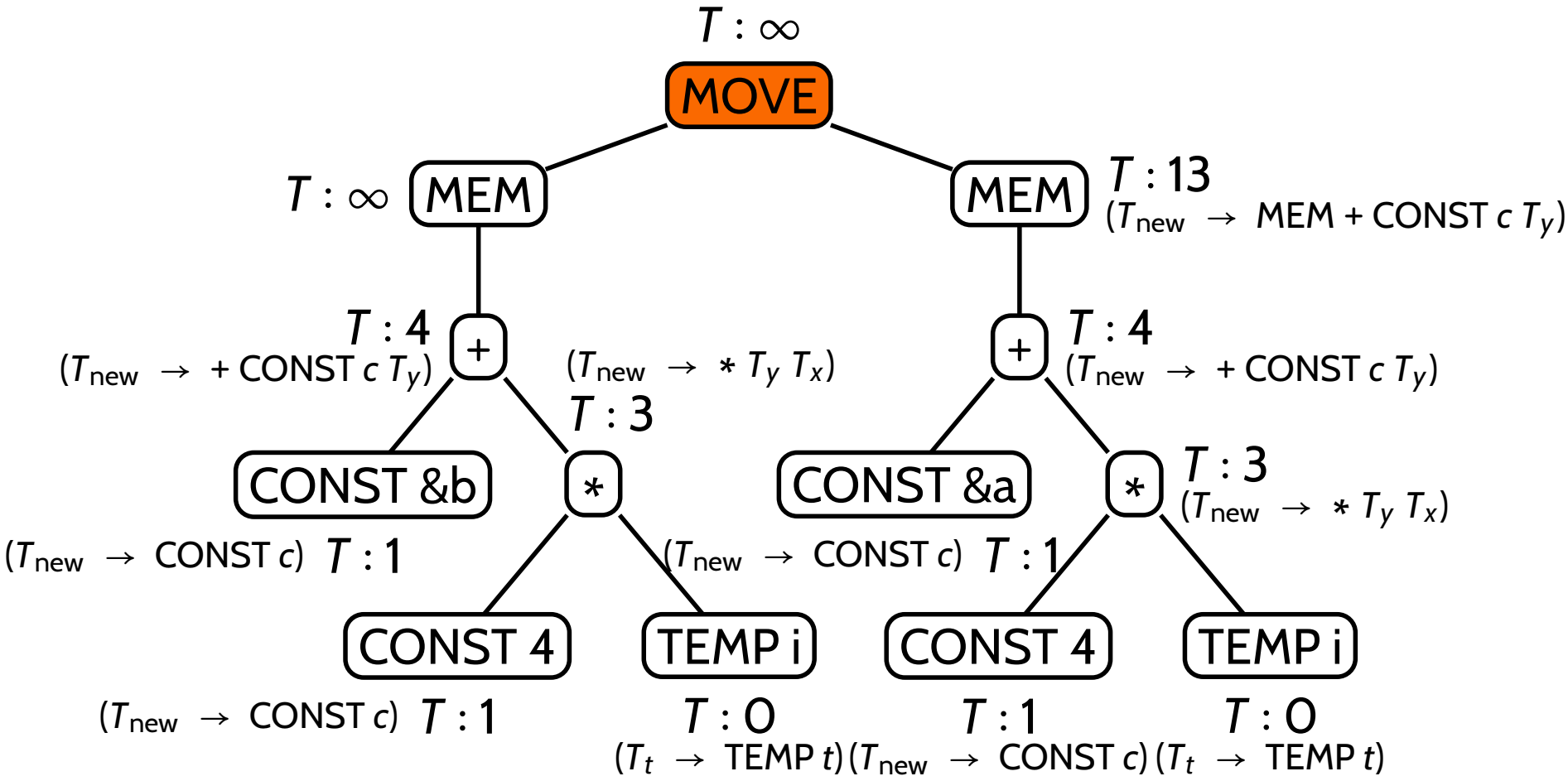
Action: compute least reduction costs

Running dynamic programming on our IR tree



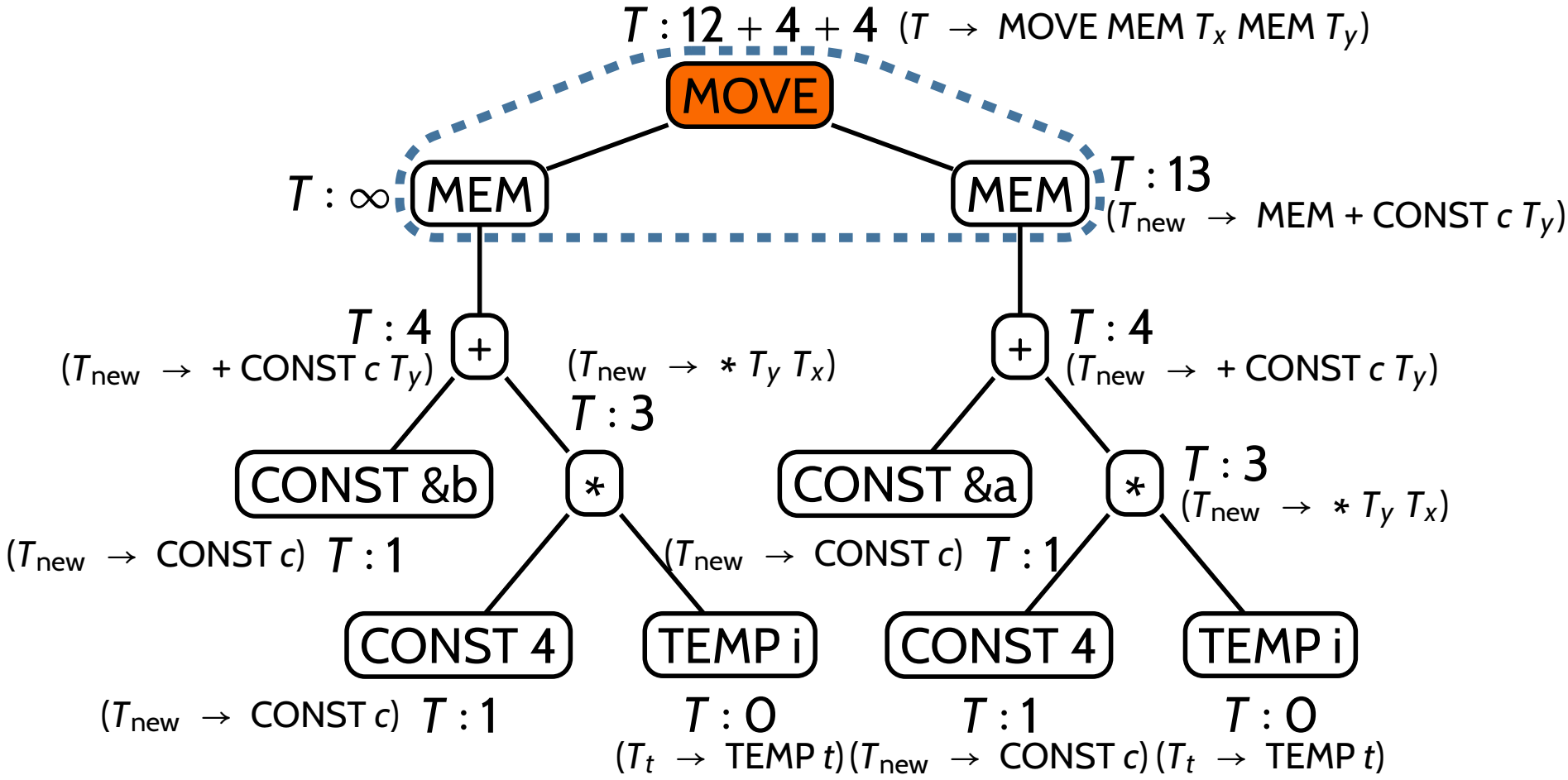
Action: compute least reduction costs

Running dynamic programming on our IR tree



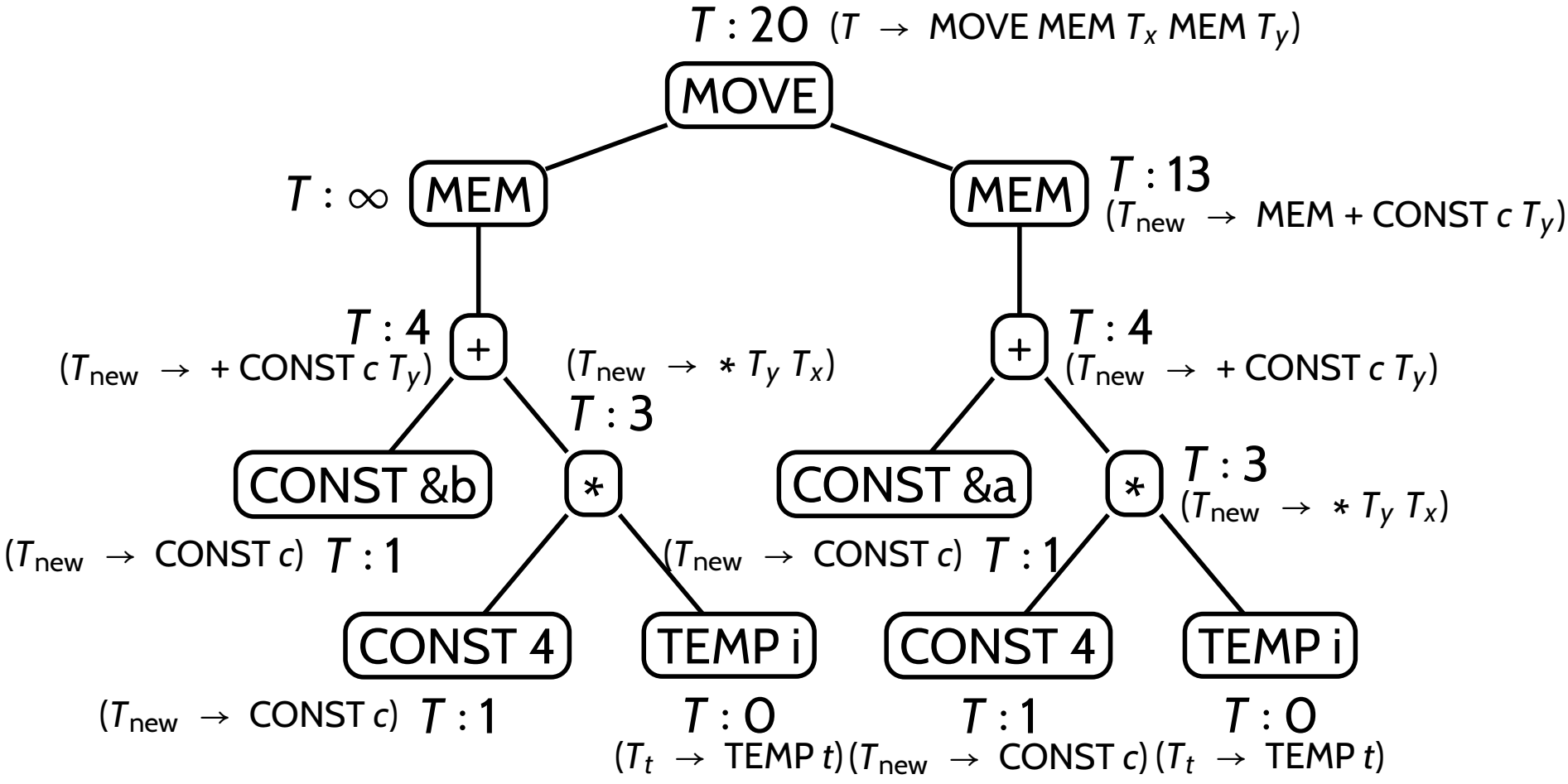
Action: compute least reduction costs

Running dynamic programming on our IR tree



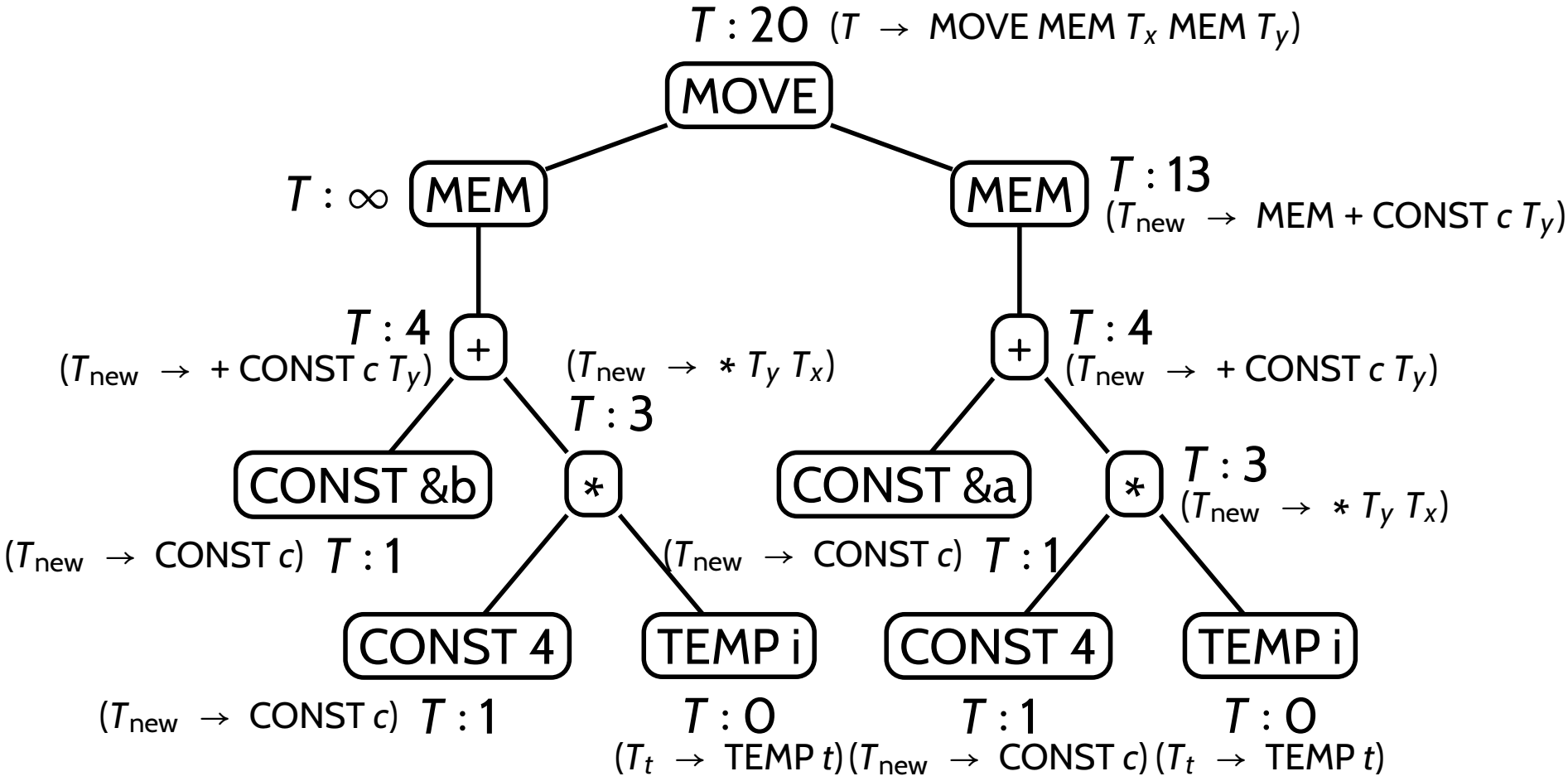
Action: compute least reduction costs

Running dynamic programming on our IR tree



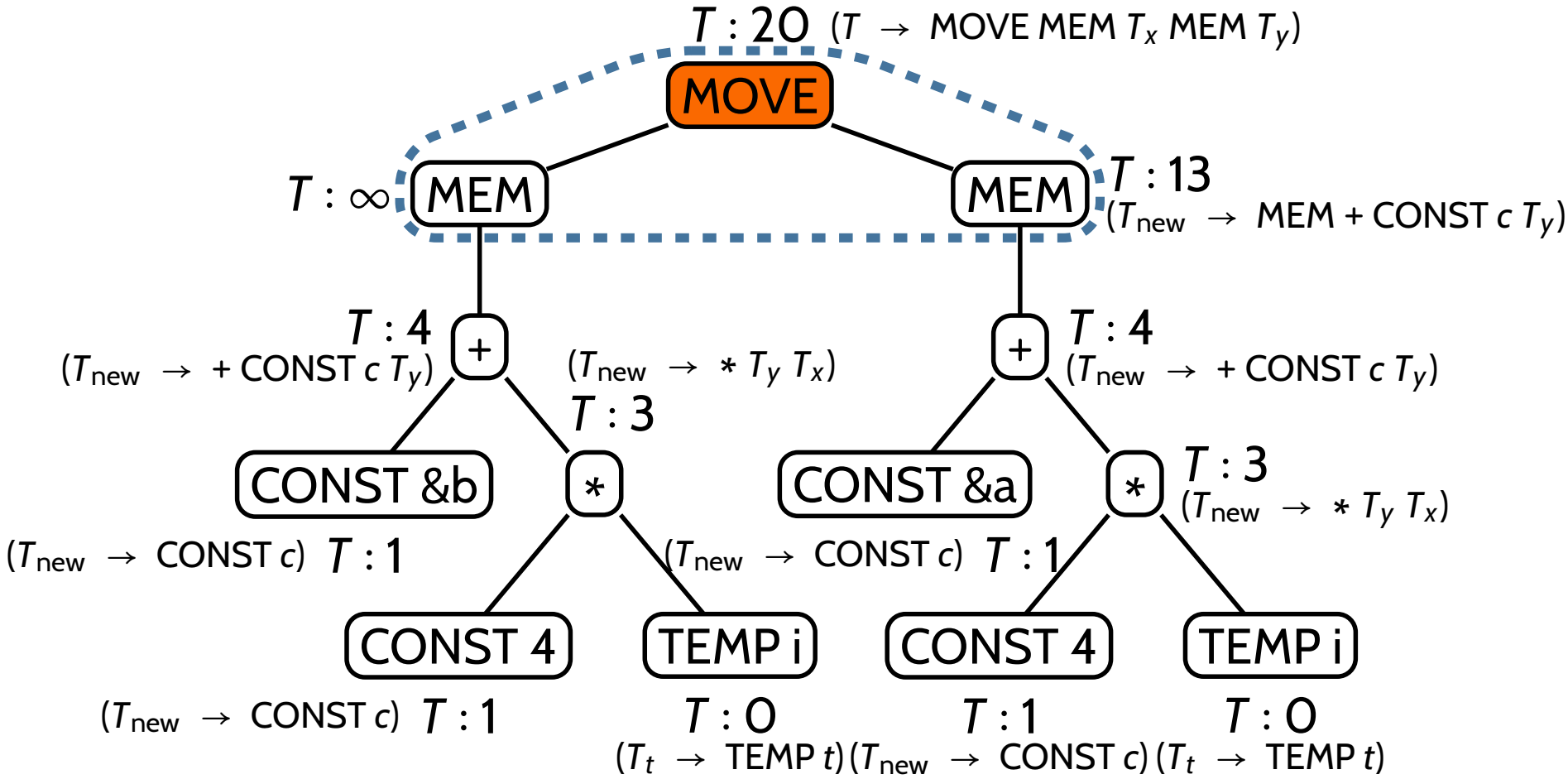
Action: done computing costs

Running dynamic programming on our IR tree



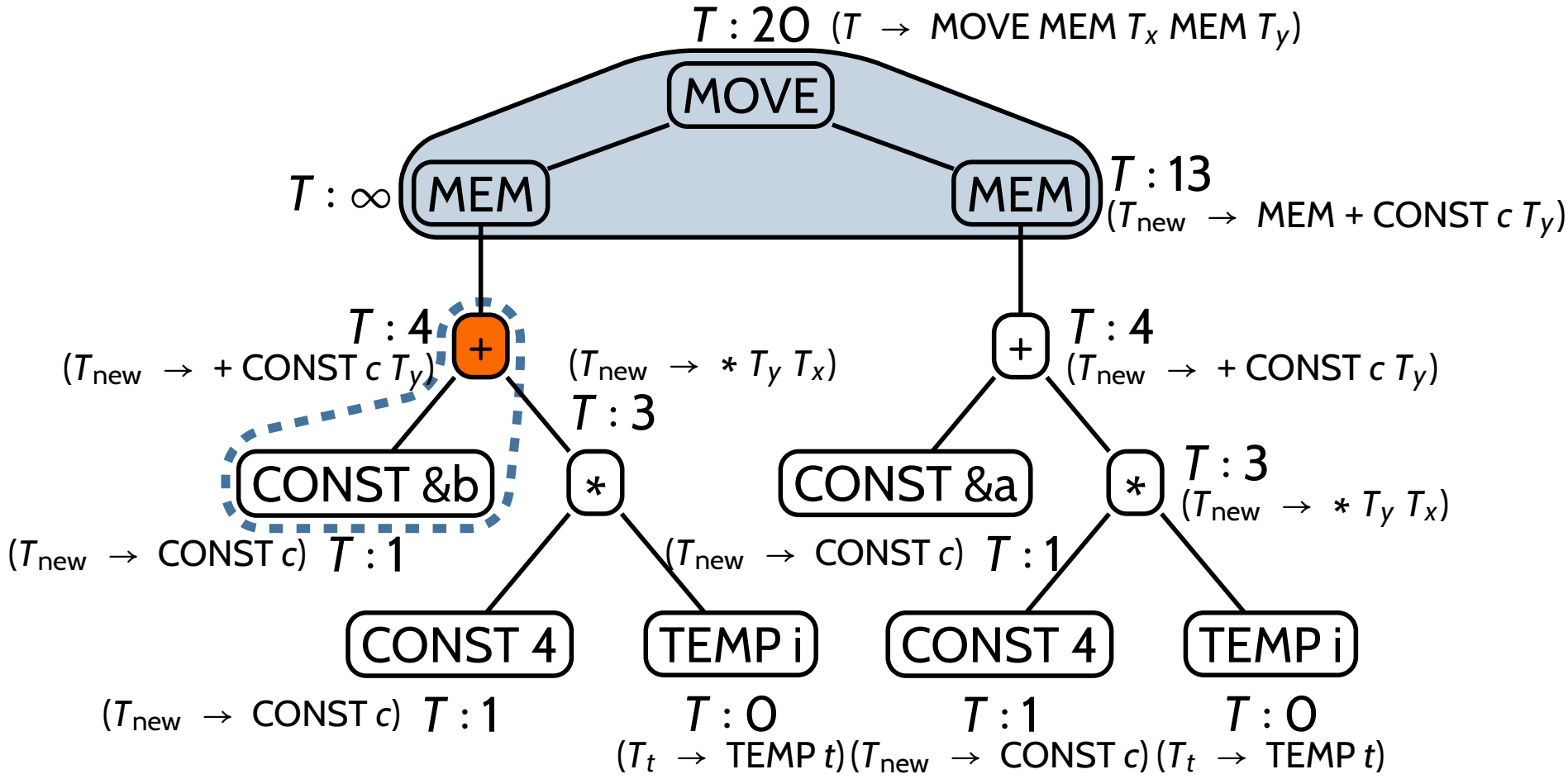
Action: select productions

Running dynamic programming on our IR tree



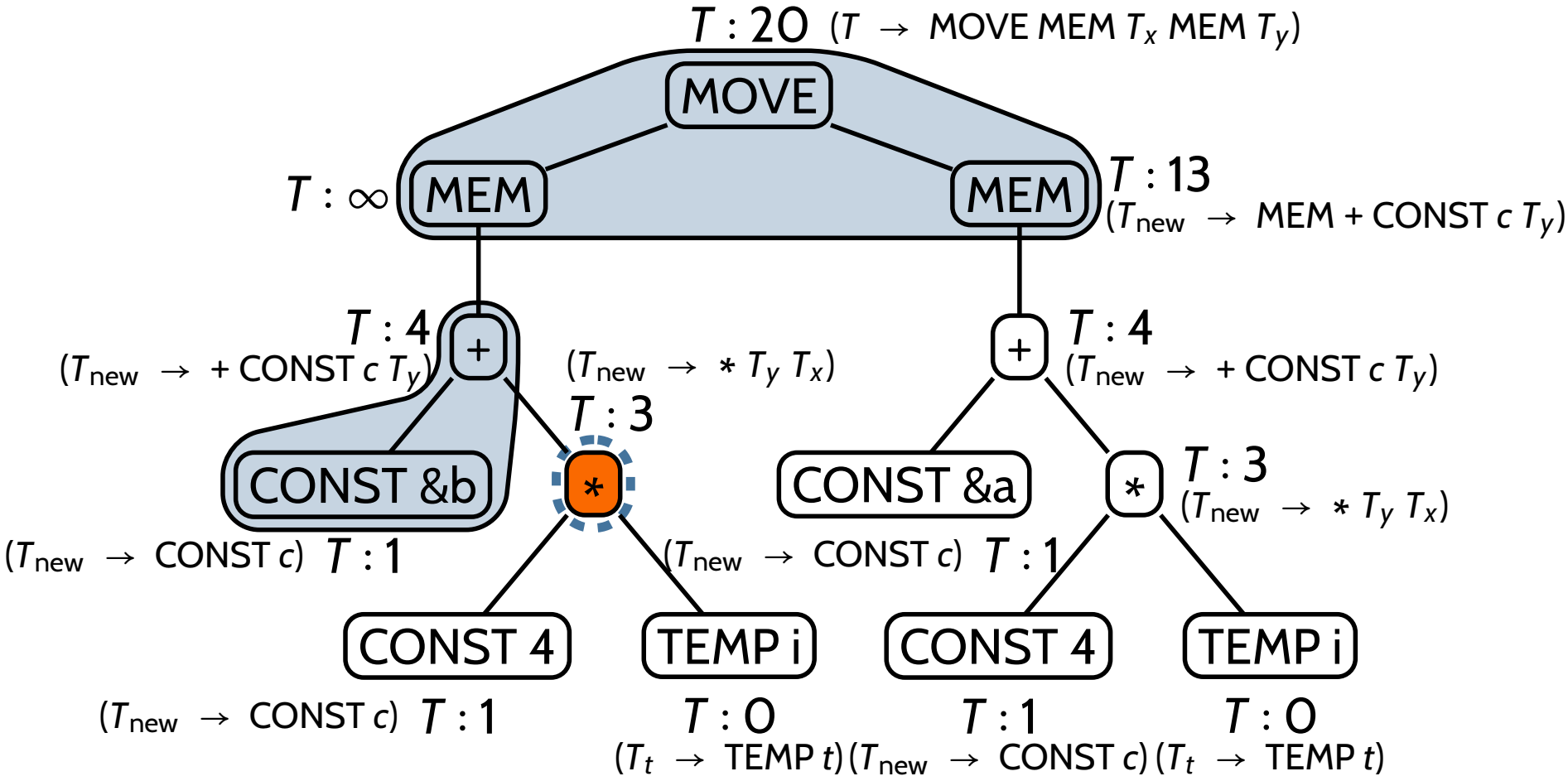
Action: select productions

Running dynamic programming on our IR tree



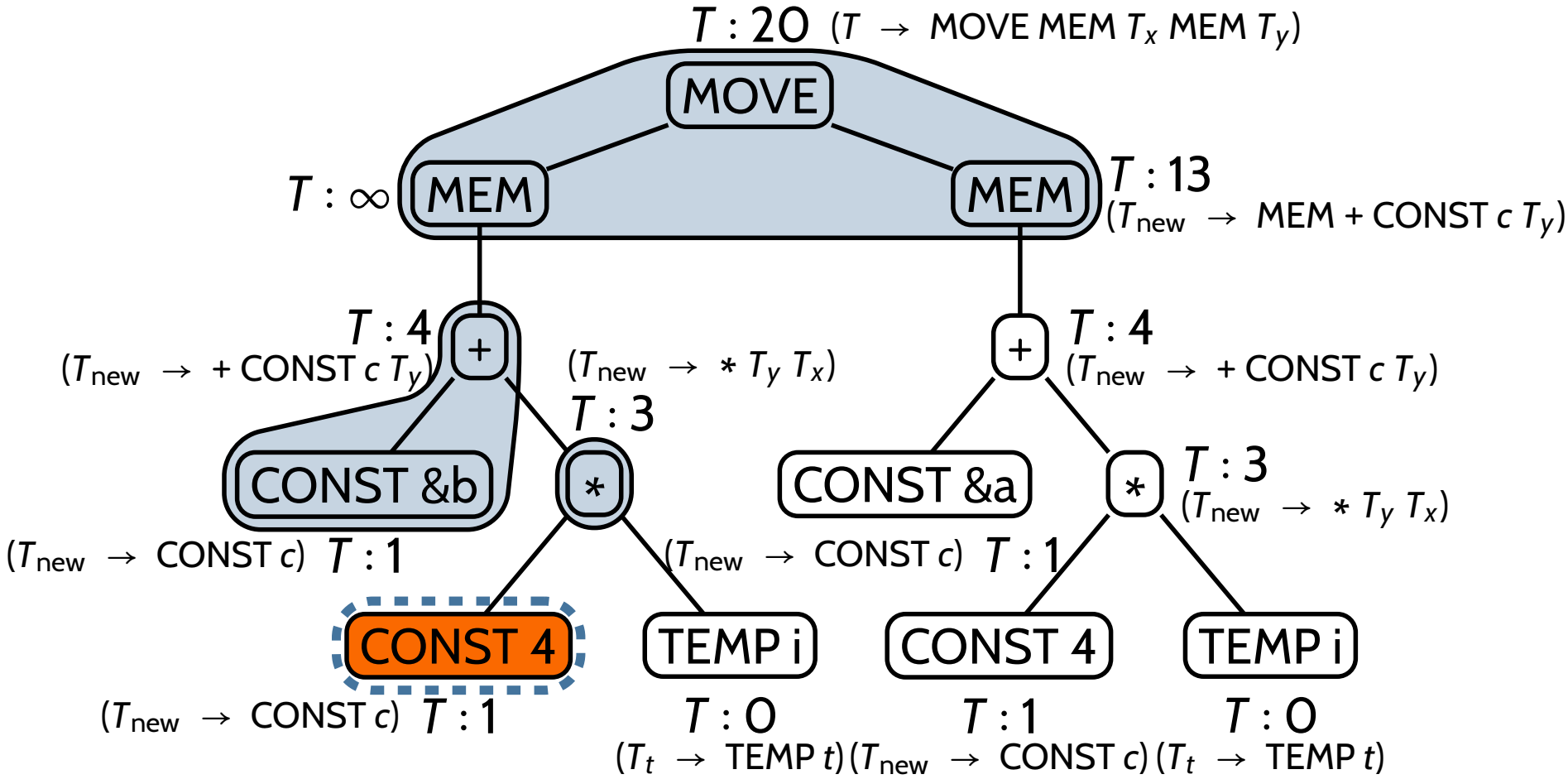
Action: select productions

Running dynamic programming on our IR tree



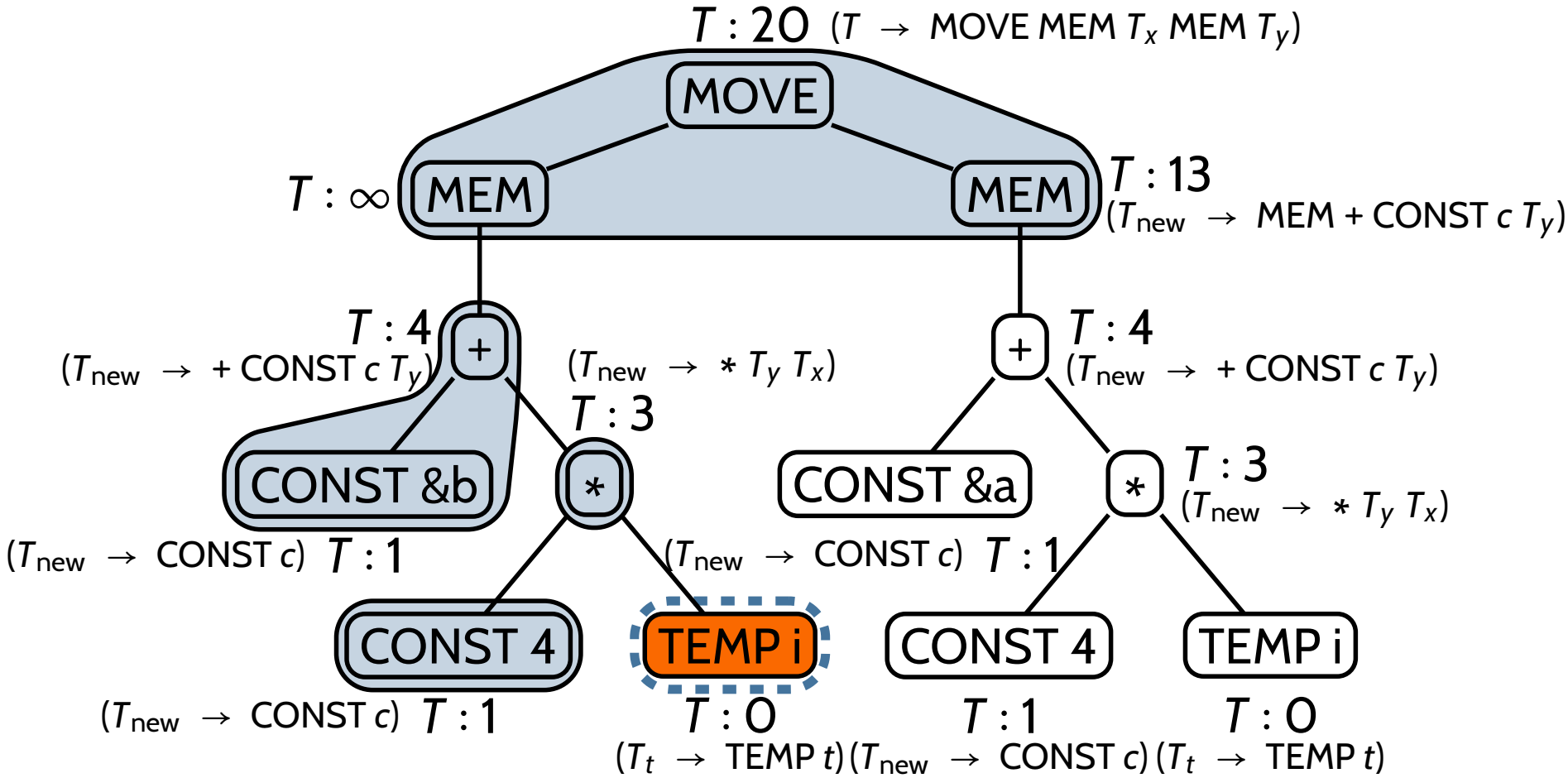
Action: select productions

Running dynamic programming on our IR tree



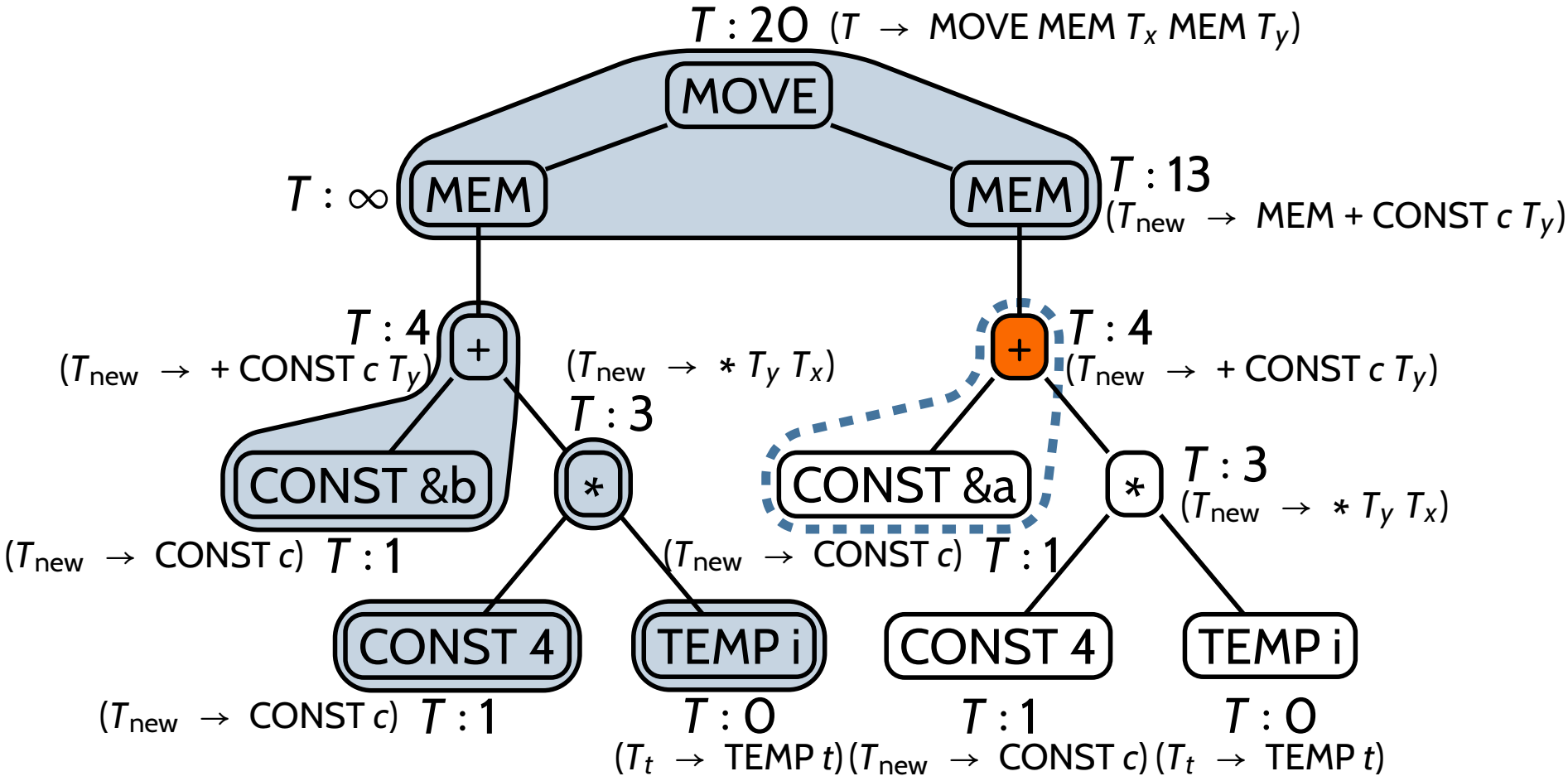
Action: select productions

Running dynamic programming on our IR tree



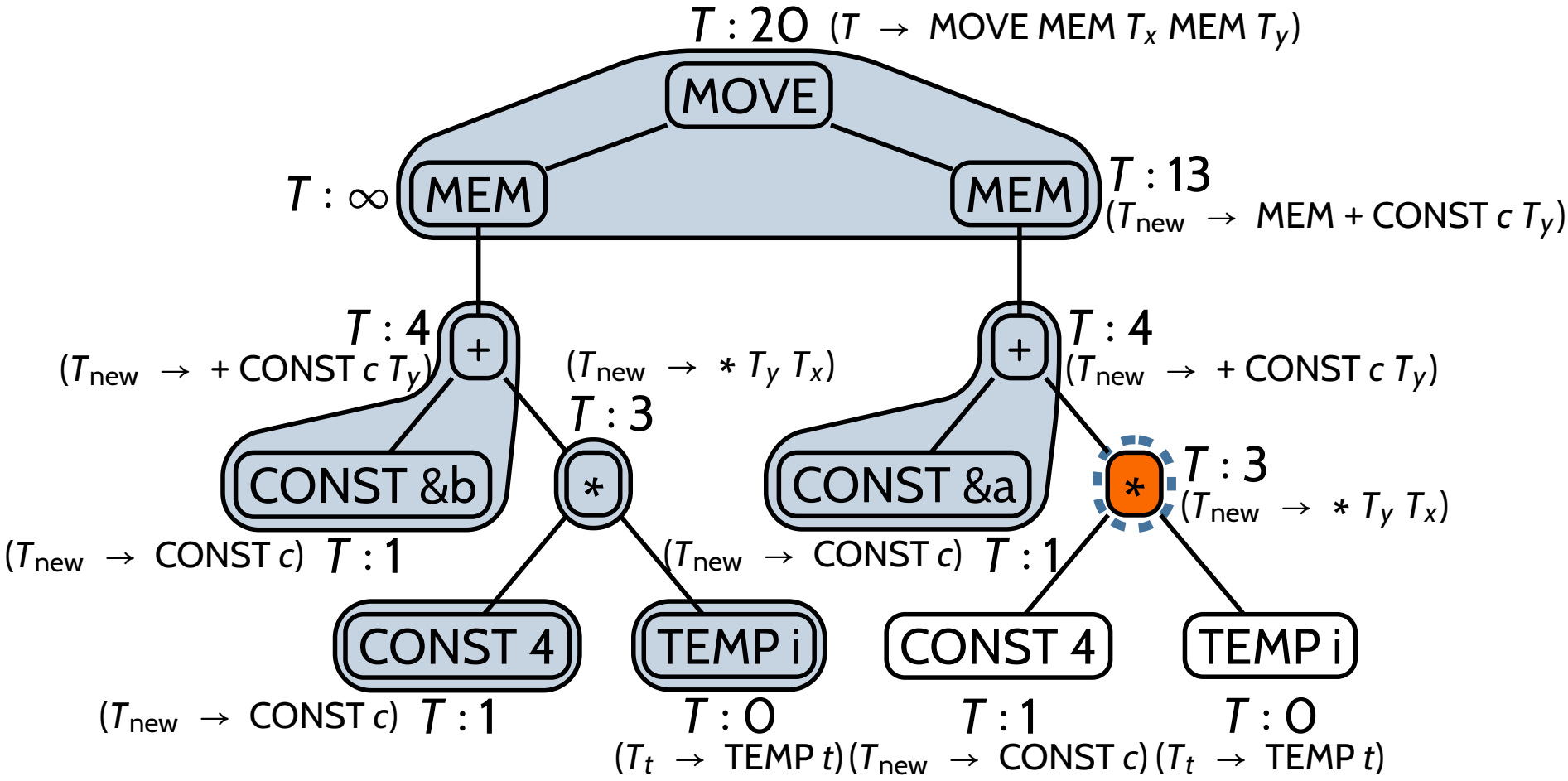
Action: select productions

Running dynamic programming on our IR tree



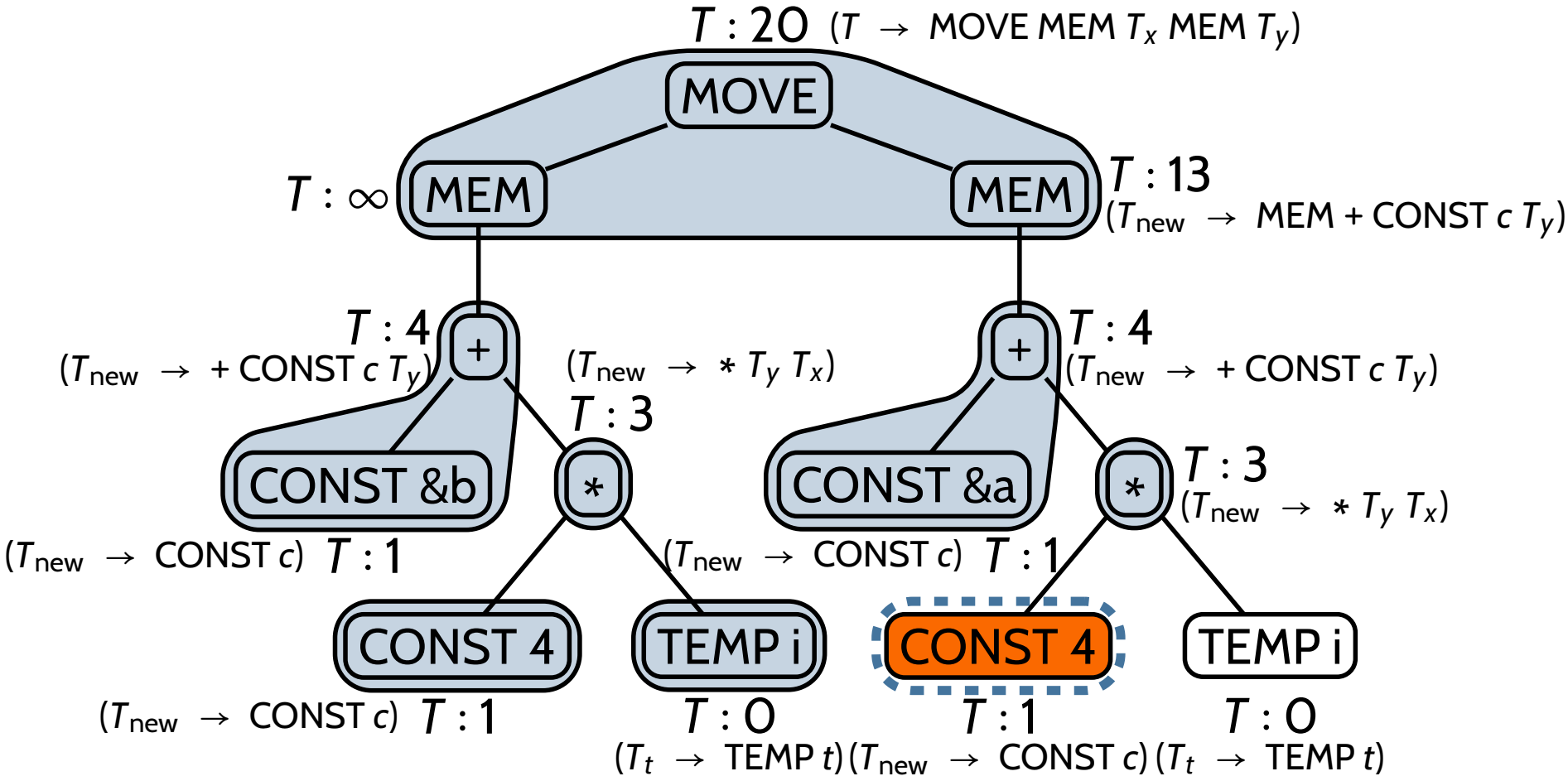
Action: select productions

Running dynamic programming on our IR tree



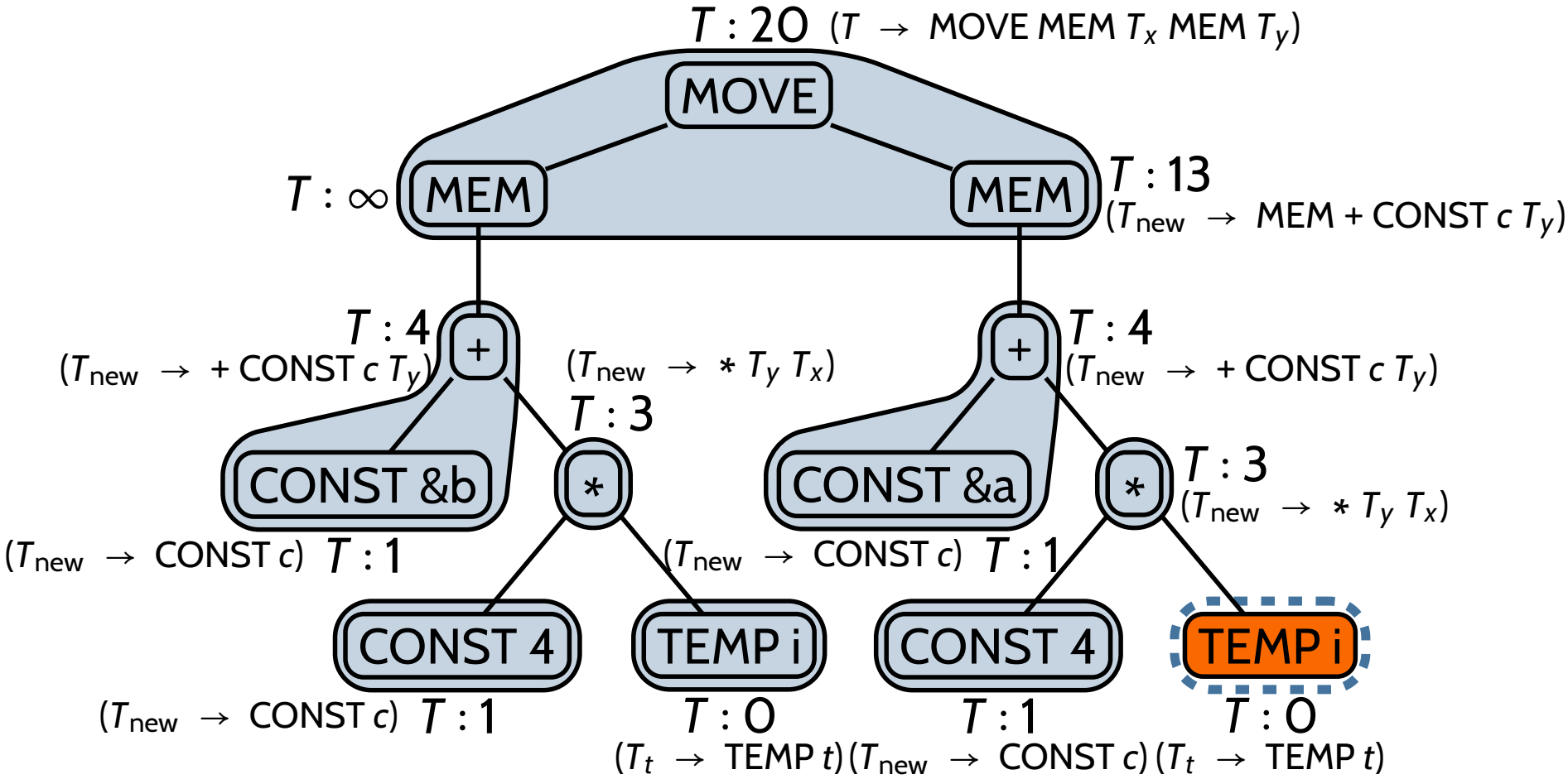
Action: select productions

Running dynamic programming on our IR tree



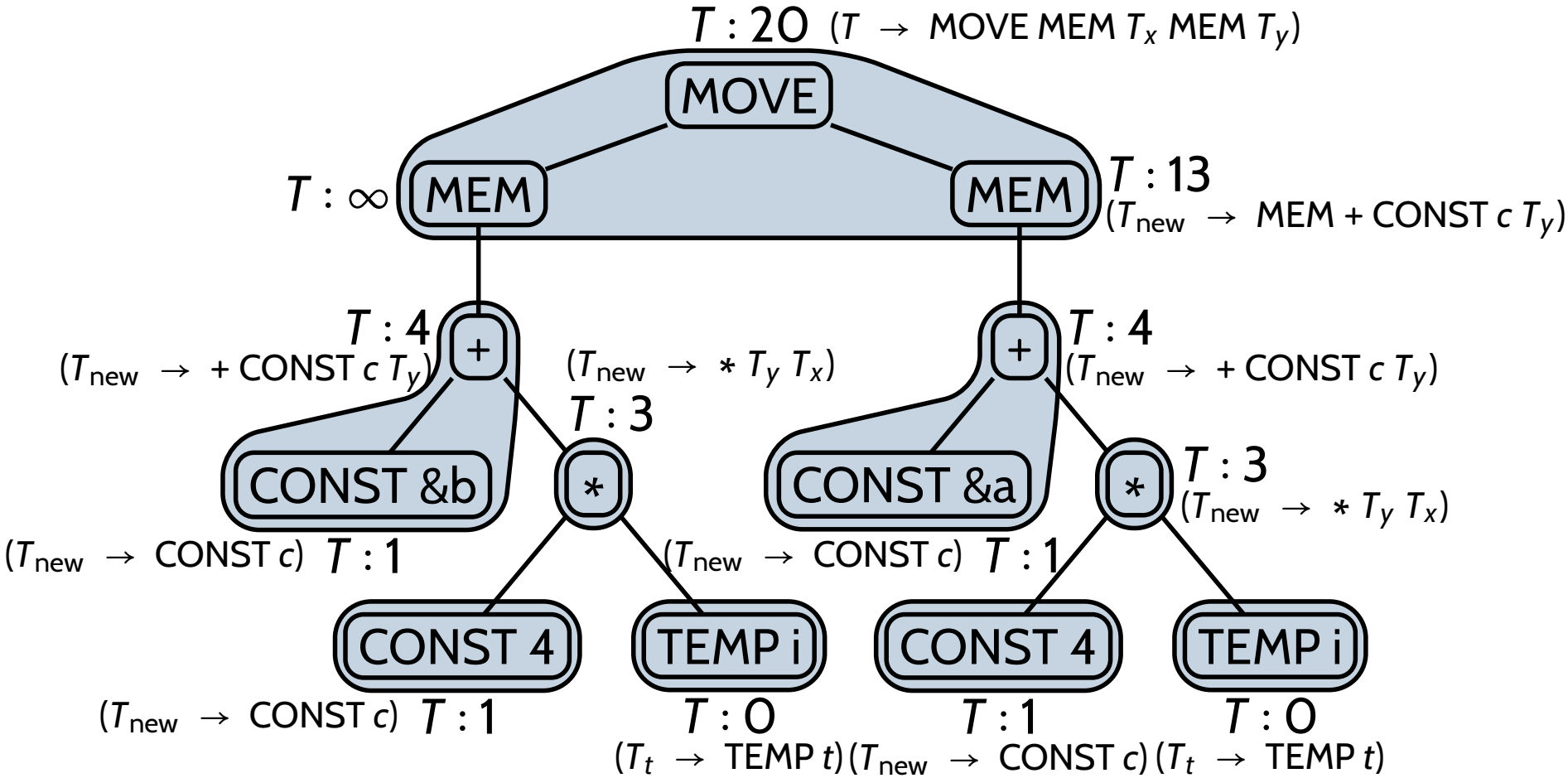
Action: select productions

Running dynamic programming on our IR tree



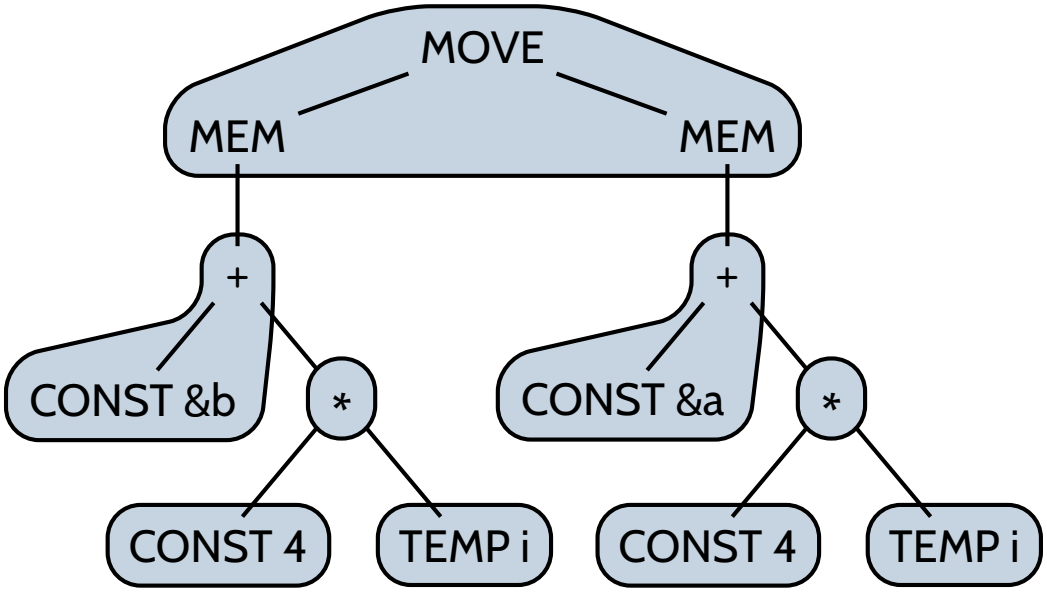
Action: select productions

Running dynamic programming on our IR tree



Action: done selecting productions

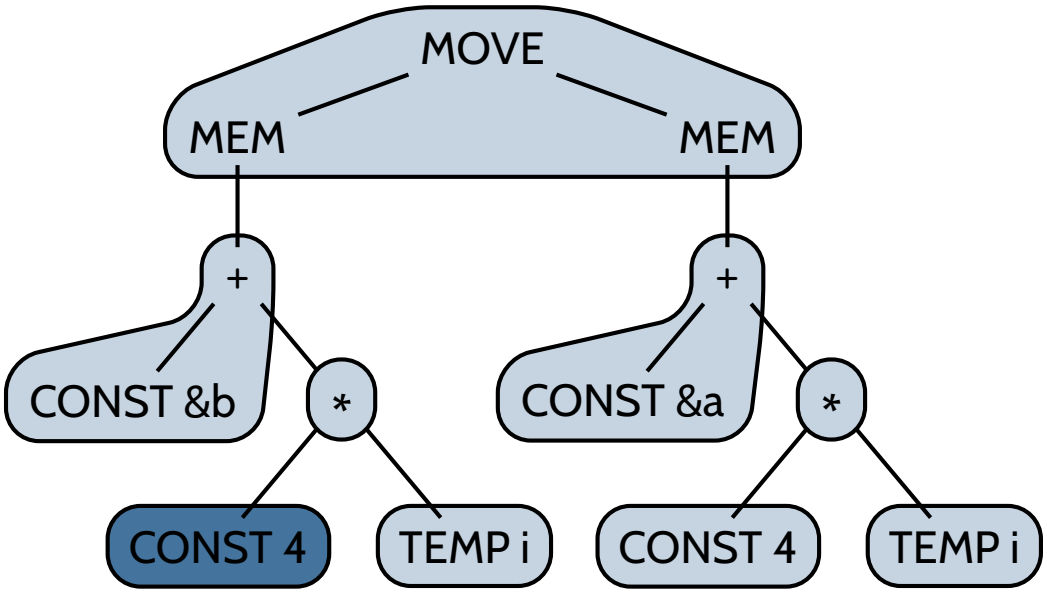
Running dynamic programming on our IR tree



Assembly code:

Action: emit assembly instructions

Running dynamic programming on our IR tree

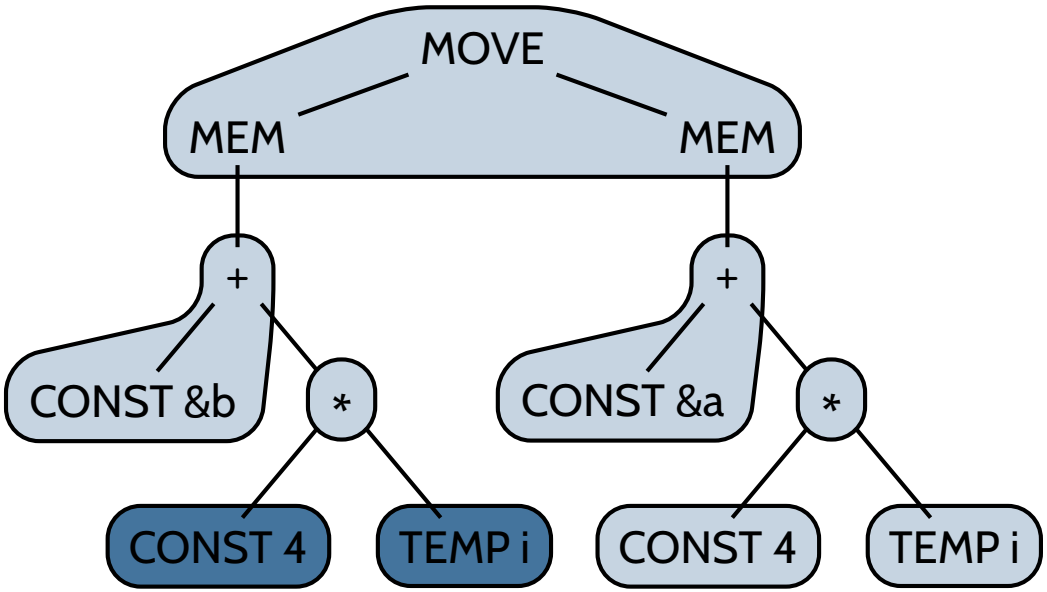


Assembly code:

ADDI $t_0 \leftarrow r_0 + \#4$

Action: emit assembly instructions

Running dynamic programming on our IR tree

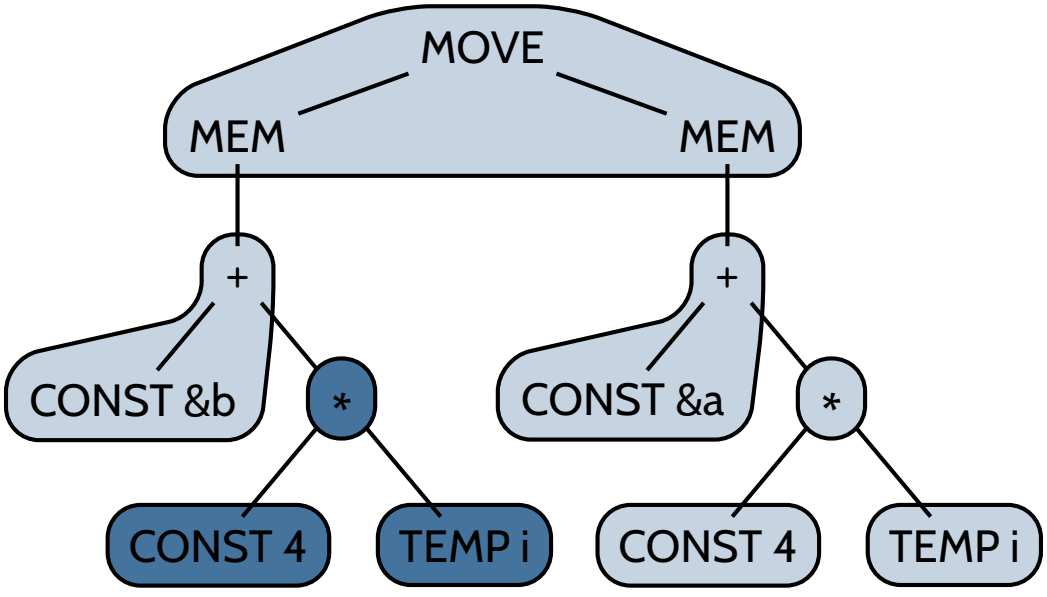


Assembly code:

ADDI $t_0 \leftarrow r_0 + \#4$

Action: emit assembly instructions

Running dynamic programming on our IR tree

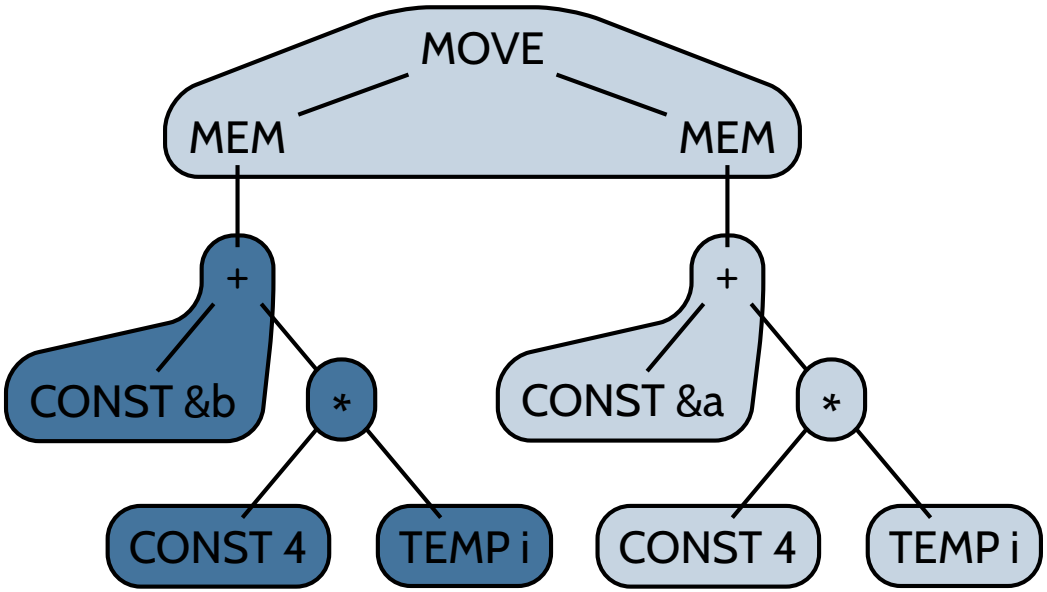


Assembly code:

```
ADDI    t0 ← r0 + #4
MUL     t1 ← t0 * ti
```

Action: emit assembly instructions

Running dynamic programming on our IR tree

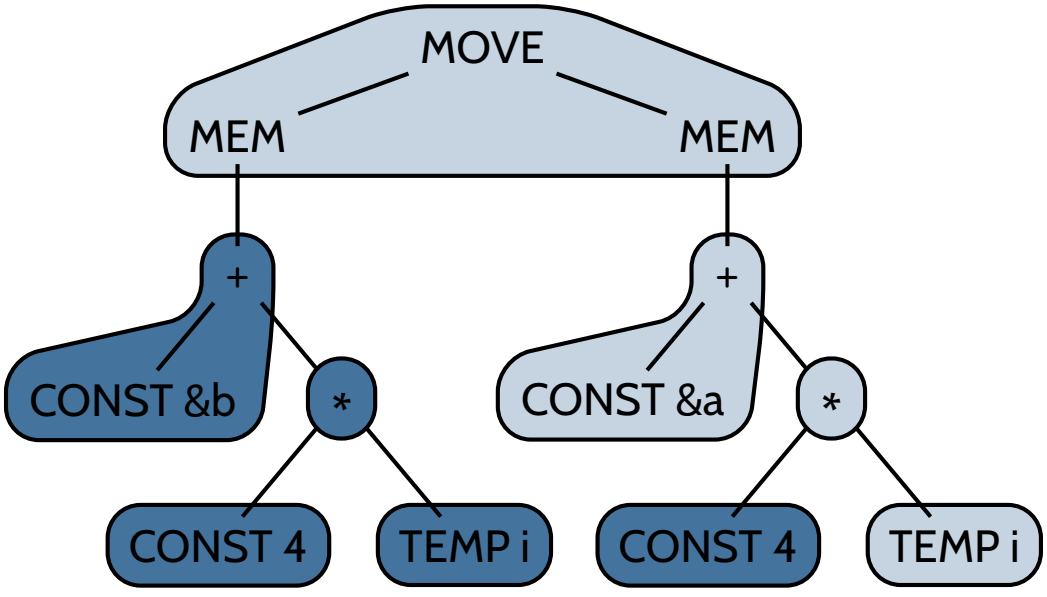


Assembly code:

```
ADDI    t0 ← r0 + #4
MUL     t1 ← t0 * ti
ADDI    t2 ← t1 + #4
```

Action: emit assembly instructions

Running dynamic programming on our IR tree

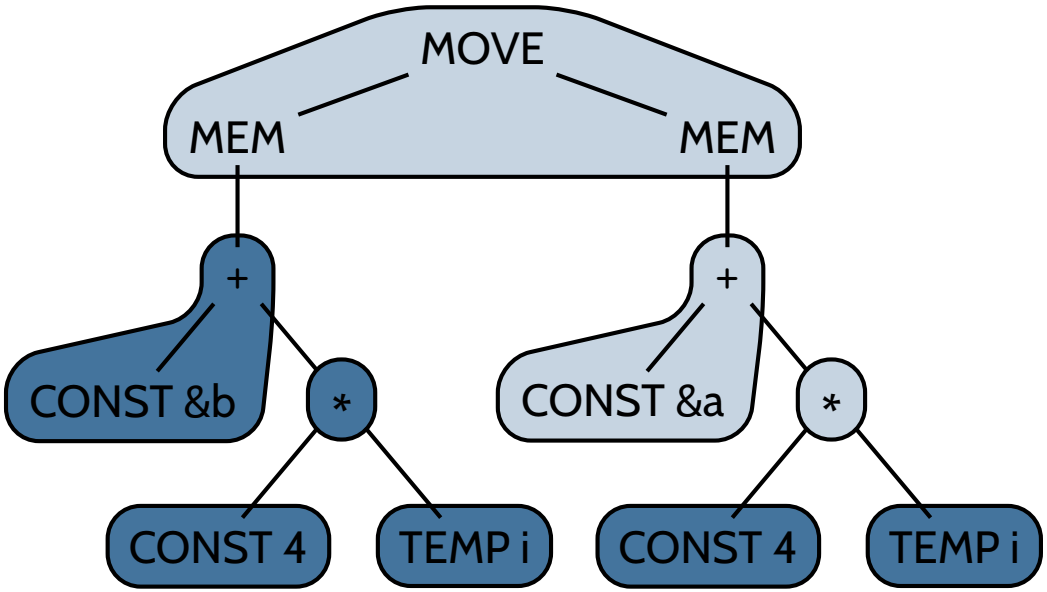


Assembly code:

```
ADDI    t0 ← r0 + #4
MUL     t1 ← t0 * ti
ADDI    t2 ← t1 + #4
ADDI    t3 ← r0 + #4
```

Action: emit assembly instructions

Running dynamic programming on our IR tree

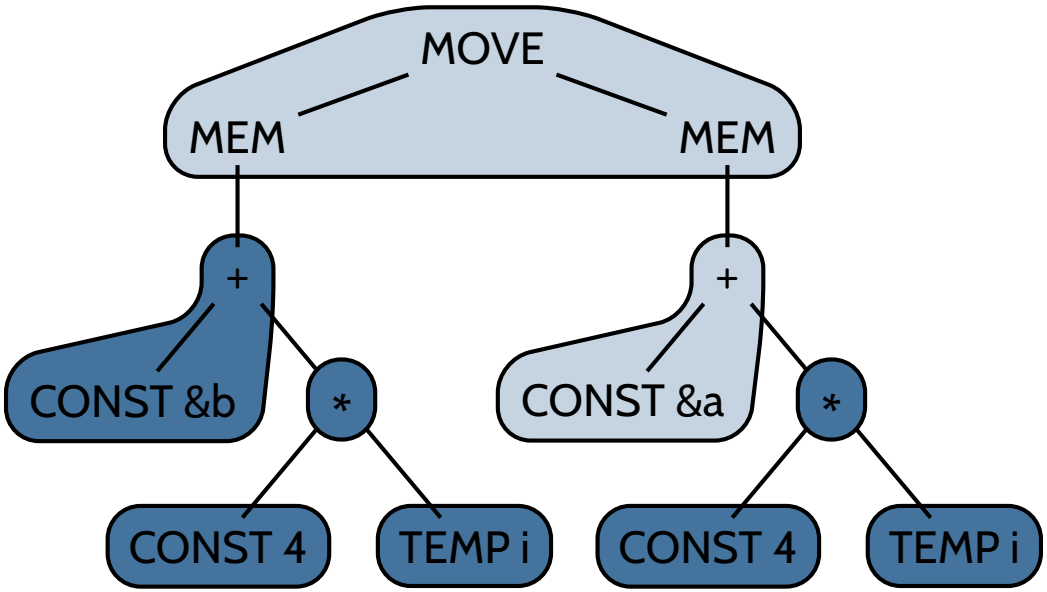


Assembly code:

```
ADDI    t0 ← r0 + #4
MUL     t1 ← t0 * ti
ADDI    t2 ← t1 + #4
ADDI    t3 ← r0 + #4
```

Action: emit assembly instructions

Running dynamic programming on our IR tree

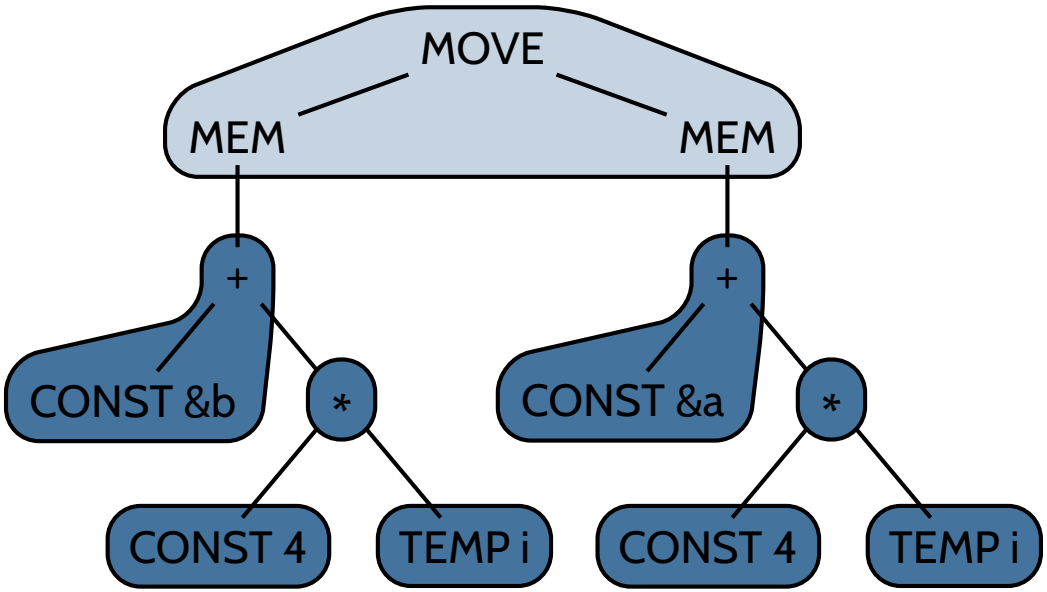


Assembly code:

```
ADDI    t0 ← r0 + #4
MUL     t1 ← t0 * ti
ADDI    t2 ← t1 + #4
ADDI    t3 ← r0 + #4
MUL     t4 ← t3 * ti
```

Action: emit assembly instructions

Running dynamic programming on our IR tree

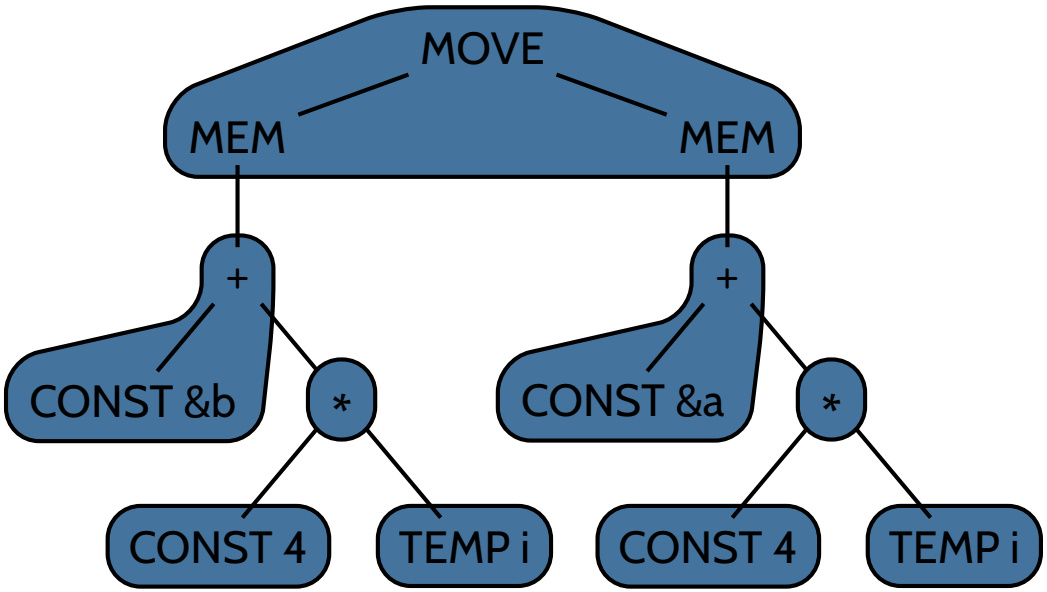


Assembly code:

```
ADDI    t0 ← r0 + #4
MUL     t1 ← t0 * ti
ADDI    t2 ← t1 + #4
ADDI    t3 ← r0 + #4
MUL     t4 ← t3 * ti
ADDI    t5 ← t4 + #4
```

Action: emit assembly instructions

Running dynamic programming on our IR tree

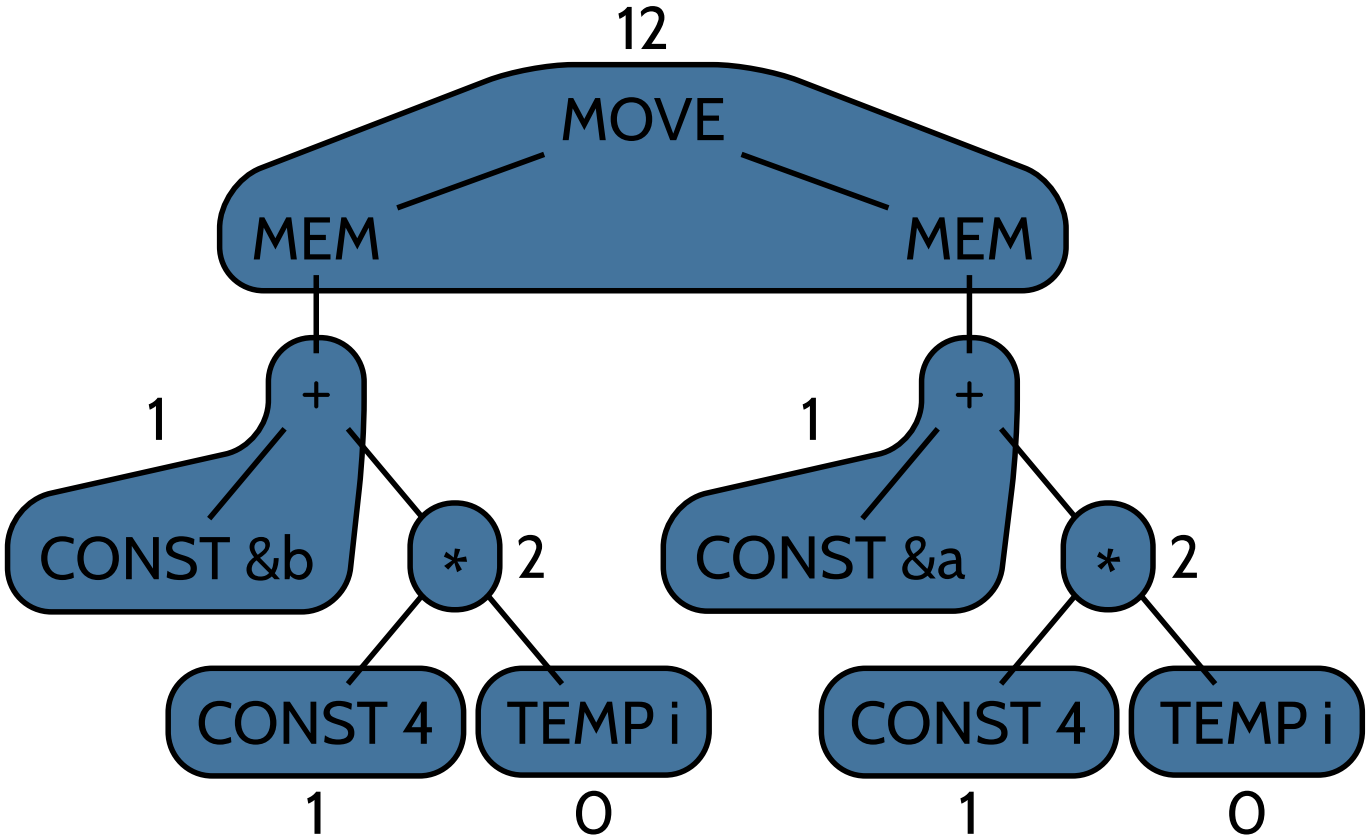


Assembly code:

```
ADDI   t0 ← r0 + #4
MUL    t1 ← t0 * ti
ADDI   t2 ← t1 + #4
ADDI   t3 ← r0 + #4
MUL    t4 ← t3 * ti
ADDI   t5 ← t4 + #4
MOVEM  M[t2] ← M[t5]
```

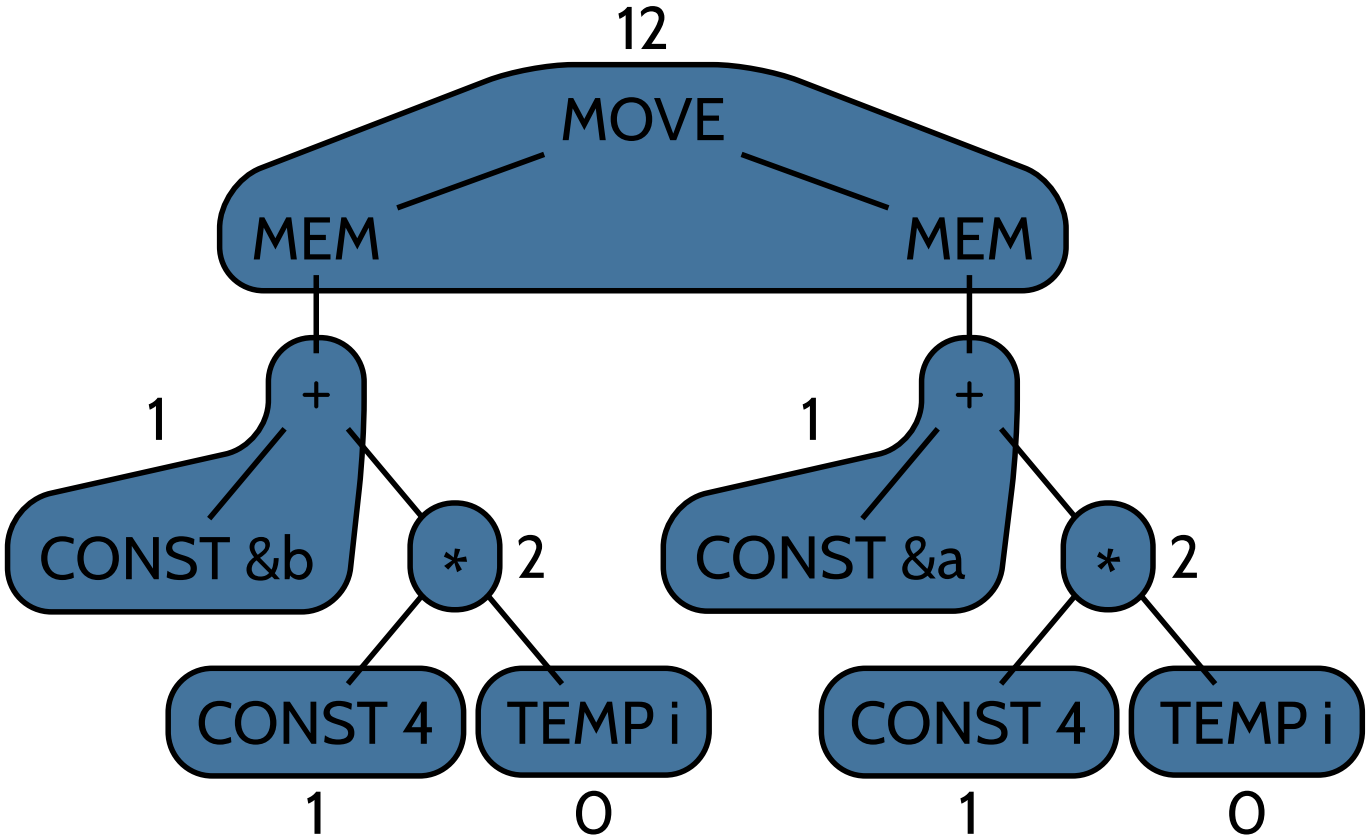
Action: done

Optimum tiling found with dynamic prog.



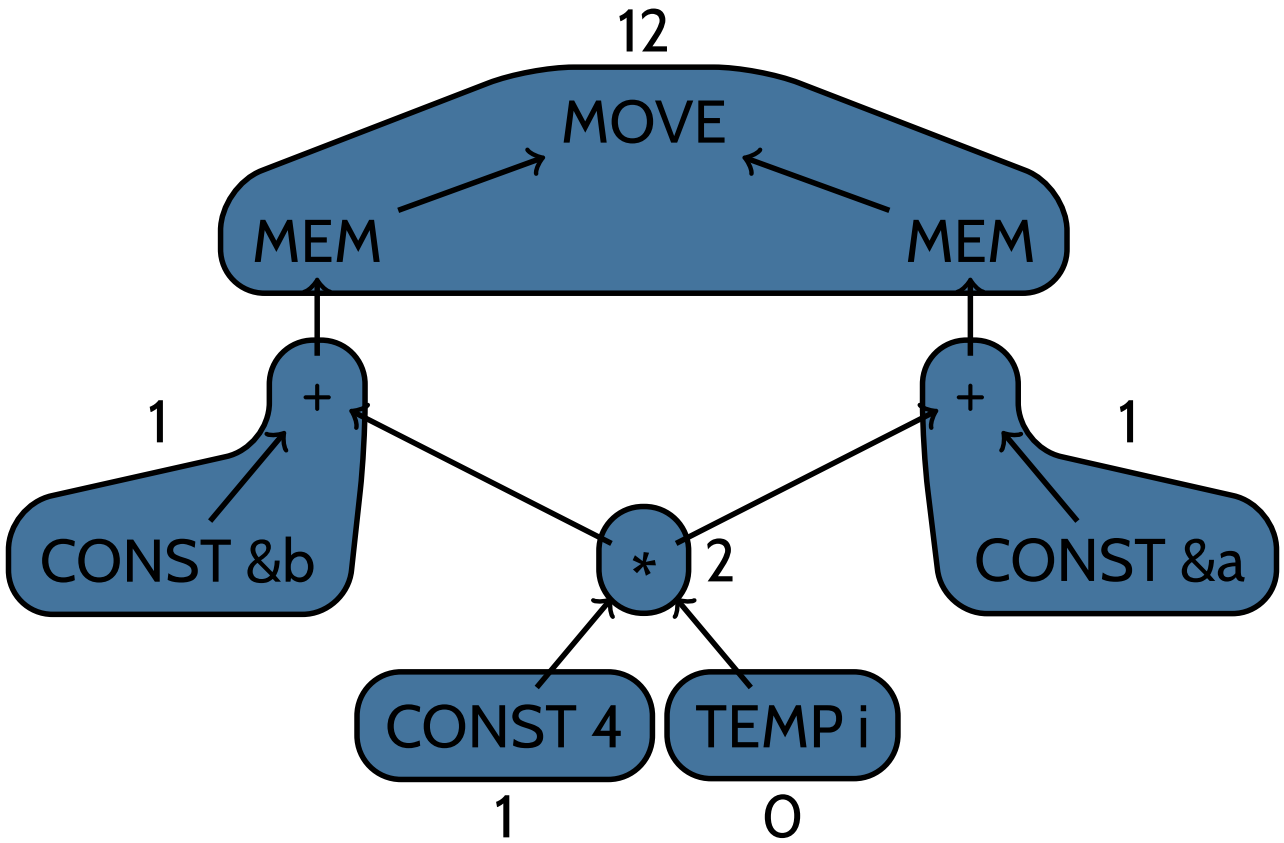
$$\sum \text{cost} = 20$$

Can we do better?



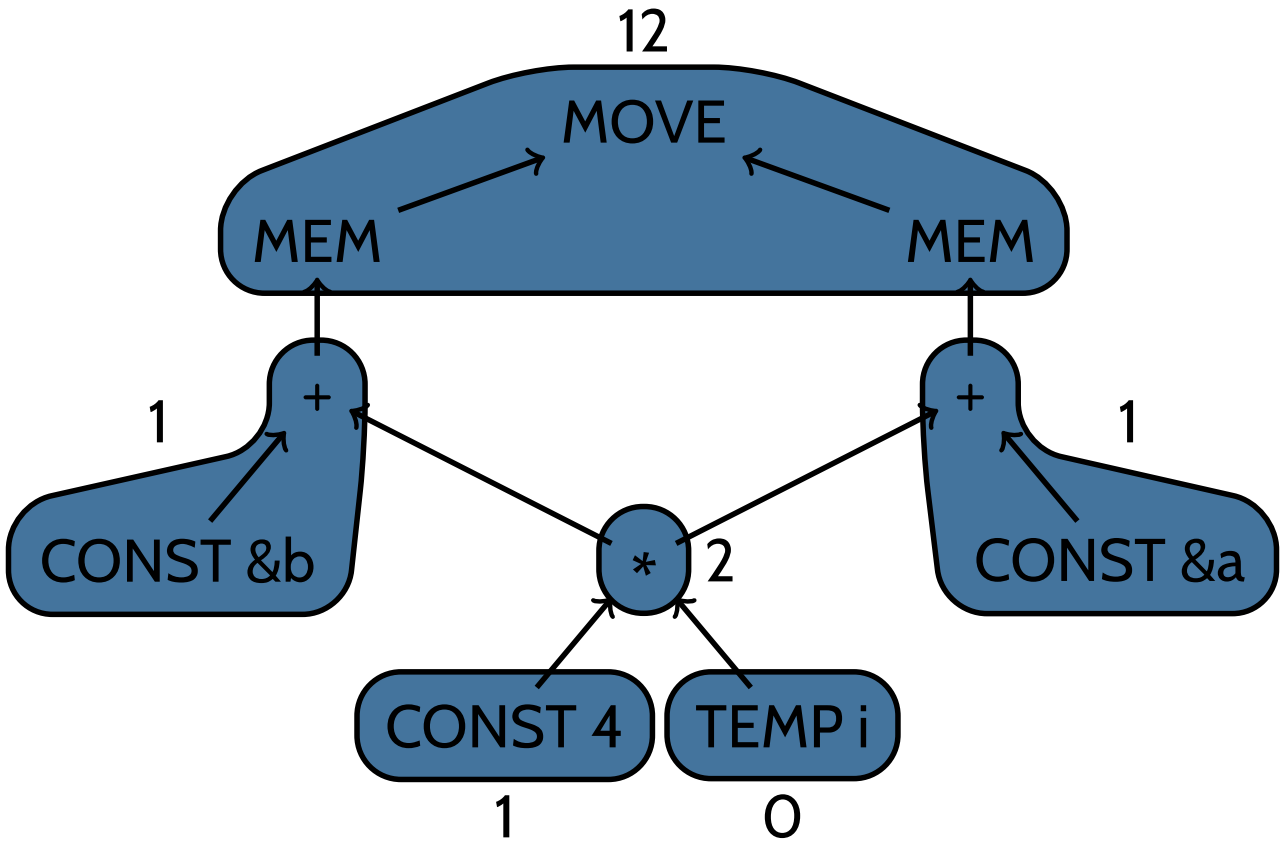
$$\sum \text{cost} = 20$$

Yes, if our IR is instead represented as a **directed acyclic graph (DAG)** . . .



$$\sum \text{cost} = 17$$

. . . but finding optimum tilings for IR DAGs is an **NP-complete** problem



$$\sum \text{cost} = 17$$

Summary

Macro expansion

- **Advantages:**
  - **Very simple** to implement
  - **Very fast** $\mathcal{O}(n)$
    - n is size of IR tree
    - Assuming tile set is fixed
- **Disadvantages:**
  - Only supports **single-node tiles**
  - May yield **suboptimal** tilings
- **Suitable for:**
  - Very simple (RISC) target architectures
    - 1-to-n mappings between IR operations and instructions
- **Modern implementations:**
  - Improved variant used in *GCC*

Maximum munch

- **Advantages over macro expansion:**
  - Supports **any-size** tree tiles
  - Always yields **optimal** tilings
  - **Very fast** $\mathcal{O}(n)$
    (provided single-node tiles exist for all IR operations)
- **Disadvantages:**
  - May yield **suboptimum** tilings
- **Suitable for:**
  - Target architectures where tile cost is **proportional** to tile size
- **Modern implementations:**
  - DAG-variant used in *LLVM*

Tree parsing

- **Advantages over maximum munch:**
  - Can **reuse** LR parsing techniques
  - **Very fast** $\mathcal{O}(n)$
- **Disadvantages:**
  - Can **fail** due to syntactic blocking
  - May still yield **suboptimum** tilings
- **Suitable for:**
  - Same as maximum munch
- **Modern implementations:**
  - None as far as I know

Dynamic programming

- **Advantages over tree parsing:**
  - Always yields **optimum** tilings
  - **Very fast** $\mathcal{O}(n)$
- **Disadvantages:**
  - More complicated compared to other approaches
  - Requires **IR trees** as input
- **Suitable for:**
  - Target architectures where all instructions are modeled as tree tiles
- **Modern implementations:**
  - *CoSy*
  - BURG —"BURGer phenomenon"→ DBURG, GBURG, GPBURG, IBURG, JBURG, HBURG, LBURG, MBURG, OCAMLBURG, and WBURG

Further reading

- **Technical report:**
  - Gabriel Hjort Blindell – *Survey on Instruction Selection: An Extensive and Modern Literature Review* (2013)
    `http://www.diva-portal.org/smash/record.jsf?pid=diva2:653943`
  - PDF downloadable for free
  - Extended version soon available as book